

Taras Shevchenko National University of Kyiv
Institute of Mathematics of National Academy of Sciences of Ukraine
National University of “Kyiv-Mohyla Academy”

**International
mathematical conference
dedicated to the 60th anniversary
of the department of algebra
and mathematical logic
of Taras Shevchenko National
University of Kyiv**

Book of Abstracts

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Department of algebra and mathematical logic of Kyiv University was created by the initiative of professor Lev Kalužnin in 1959. He was the first head and scientific founder of the department. Tsilya Shub and Volodymyr Vyshensky were the first employees. In later years the department was headed by professors Sergiy Zavalo, Yuriy Drozd, Vitaly Sushchansky and Anatoliy Petravchuk, who heads the department at the present time. During the past 60 years professors Volodymyr Bavula, Victor Charin, Vyacheslav Futorny, Volodymyr Kirichenko, Ganna Kudryavtseva, Volodymyr Nekrashevych, Volodymyr Mazorchuk, Sergiy Ovsienko, Volodymyr Sergeichuk, Vasyl Ustimenko worked at the department.

Research topics of the department are broad and include different areas of pure and applied algebra. Group and semigroup theory, geometric and asymptotic group theory, Lie algebras, matrix algebras, representation theory, cryptography and combinatorics are currently in focus of interests at the department.

Department of algebra and mathematical logic maintains scientific collaboration with many brilliant researchers all over the world. We are proud of our alumni, colleagues, friends and teachers. We always welcome you to Kyiv.

Planned as a classical mathematical conference due to circumstances our meeting changes format. For most of us the current time is challenging. We wish everyone good health and many new interesting results.

Organizing Committee

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Local nearrings of order 1331

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Nearrings are generalization of associative rings, in which the additive group can be non-Abelian, and addition is connected with multiplication by only one distributive law, left or right. In this sense local nearrings are generalization of local rings. The package SONATA [2] of the computer algebra system GAP [1] contains a library of all non-isomorphic nearrings of order at most 15 and nearrings with a unity of order up to 31, among which 698 are local. However, the classification of nearrings of higher orders requires much more complex calculations. For local nearrings they were realized by us in the form of a new GAP package called LocalNR [3]. Its current version (not yet distributed with GAP) contains 37599 local nearrings of order at most 361, except orders 128, 256 and some of orders 32, 64 and 243. We have already calculated some classes of local nearrings of orders 32 (with 14927685 nearrings), 64 (with 1115947 nearrings) and 243 (with 705105 nearrings). It is known that there are 5 non-isomorphic groups of order $11^3 = 1331$. It turns out that all of them are the additive groups of local nearrings. The following table contains the list of all non-isomorphic nearrings of this order.

Theorem 1. *There exist 158 local nearrings of order 1331:*

Additive groups	Number of local nearrings
C_{1331}	1
$C_{121} \times C_{11}$	47
$(C_{11} \times C_{11}) \rtimes C_{11}$	12
$C_{121} \rtimes C_{11}$	2
$C_{11} \times C_{11} \times C_{11}$	96

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- [3] Raievska I., Raievska M., Sysak Y., LocalNR, Package of local nearrings, Version 1.0.2, 2020. (GAP package), (<https://gap-packages.github.io/LocalNR>).

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O. D. Artemovych

Non-associative structures of non-commutative associative algebras

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In the present talk, I will discuss on non-associative algebras induced on a non-commutative associative algebra by some its operators.

Note that non-associative structures (Zinbel, Lie, Balinsky-Novikov) on a commutative associative algebra, in particular, have been studied in [1–3].

- [1] O.D. Artemovych, D. Blackmore, A.K. Prykarpatski, Non-associative structures of commutative algebras related with quadratic Poisson brackets, *European J. Math.* **6** (2020), 208–231; <https://doi.org/10.1007/s40879-020-00398-w>
- [2] O.D. Artemovych, A.A. Balinsky, D. Blackmore, A.K. Prykarpatski, Reduced pre-Lie algebraic structures, the weak and weakly deformed Balinsky-Novikov type symmetry algebras and related Hamiltonian operators, *Symmetry* **10** (2018), 601, 28 pp.; doi:10.3390/sym10110601
- [3] O.D. Artemovych, A.K. Prykarpatski, D.L. Blackmore, Examples of Lie and Balinsky-Novikov algebras related to Hamiltonian operators, *Topol. Algebra Appl.* **6** (2018), 43–52; <https://doi.org/10.1515/taa-2018-0005>

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Derivations of group rings

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Let $R[G]$ be the group ring of a group G over an associative ring R with unity such that all prime divisors of orders of elements of G are invertible in R . If R is finite and G is a Chernikov (torsion FC -) group (see [1]), then each R -derivation of $R[G]$ is inner. Similar results also are obtained for other classes of groups G and rings R .

[1] O. Artemovych, V. Bovdi and M. Salim. *Derivation on group rings*. Acta Math. Szeged, 86 (2020), P. 51–72.

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Topology in arithmetics: Golomb and Kirch topologies on \mathbb{N}

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The Golomb (resp. Kirch) topology on the set \mathbb{N} of positive integers is generated by the base consisting of the arithmetic progressions $\mathbb{N} \cap (a + b\mathbb{Z})$ where $a, b \in \mathbb{N}$ are coprime numbers (and b is square-free). It is known that the space \mathbb{N} endowed with the Golomb (resp. Kirch) topology is connected, Hausdorff (and locally connected).

Theorem 1. *The space \mathbb{N} endowed with the Golomb (resp. Kirch) topology is topologically rigid, i.e., has trivial homeomorphism group.*

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- [2] T. Banakh, D. Spirito, S. Turek, *The Golomb space is topologically rigid*, preprint (arxiv.org/abs/1912.01994).
- [3] T. Banakh, Ya. Stelmakh, S. Turek, *The Kirch space is topologically rigid*, preprint (arxiv.org/abs/2006.12357).

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Serhii Bardyla

On semitopological graph inverse semigroups

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All topological spaces are assumed to be Hausdorff. We prove that each non-zero element of a semitopological graph inverse semigroup is isolated. We characterized graph inverse semigroups which admit only discrete locally compact semigroup topology. This characterization gives an answer to the question of Mesyan, Mitchell, Morayne and Péresse. It is proved that a locally compact semitopological graph inverse semigroup over a strongly connected graph which contains finitely many vertices is either compact or discrete. We construct a universal object in the class of graph inverse semigroups. It is proved that for each \mathcal{D} -class D of a graph inverse semigroup over an acyclic graph the semigroup $D_0 = D \cup \{0\}$ is isomorphic to a semigroup of matrix units. Finally, we describe graph inverse semigroups which are subsemigroups of compact-like topological semigroups. More precisely, we characterized graph inverse semigroups which admit compact semigroup topology and graph inverse semigroups which embed densely into feebly compact topological semigroups.

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**On the lattice of weak topologies on the bicyclic monoid
with adjoined zero**

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A Hausdorff topology τ on the bicyclic monoid with adjoined zero \mathcal{C}^0 is called *weak* if it is contained in the coarsest inverse semigroup topology on \mathcal{C}^0 . We show that the lattice \mathcal{W} of all weak shift-continuous topologies on \mathcal{C}^0 is isomorphic to the lattice $SLF^1 \times SLF^1$ where SLF^1 is a set of all shift-invariant filters on ω with an attached element 1 endowed with the following partial order: $\mathcal{F} \leq \mathcal{G}$ iff $\mathcal{G} = 1$ or $\mathcal{F} \subset \mathcal{G}$. Also, we investigate cardinal characteristics of the lattice \mathcal{W} . In particular, we proved that \mathcal{W} contains an antichain of cardinality $2^{\mathfrak{c}}$ and a well-ordered chain of cardinality \mathfrak{c} . Moreover, there exists a well-ordered chain of first-countable weak topologies of order type \mathfrak{t} .

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Leonid Bedratyuk

Visual pattern recognition and invariant theory

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The orientation-preserving transformation group G is widely used in 2D image analysis and pattern recognition. The introduction of the notion of image moment G -invariants by Hu was a vivid example of the application of the classical invariant theory to the pattern recognition. Let \mathbf{F} be a set of real finite piecewise continuous functions (*digital images*) that can have nonzero values only in a compact subset of \mathbb{R}^2 . Hu have considered *the geometric moments* $m_{pq}(f)$ of $f \in \mathbf{F}$, the *central geometric moment* $\mu_{pq}(f)$

$$m_{pq}(f(x, y)) = \iint_{\Omega} x^p y^q f(x, y) dx dy, \Omega \subset \mathbb{R}^2,$$

the normalized moments $\eta_{pq} := \eta_{pq}(f)$ which are invariants of the uniform scaling group. The problem of determining G -invariants can be reduced to problem of finding $SO(2)$ -invariants as functions of the normalized central geometric moments.

Denote by $\mathbb{C}(\eta_d)^{SO(2)}$ where $\eta_d = \{\eta_{p,q}, 2 \leq p + q \leq d\}$ the corresponding algebra of the *simultaneous* 2D geometric moment invariants of *order* up to d . Remarkably, the problem of describing of the algebras simultaneous 2D geometric moment invariants in the general case can be reduced to well-known problems of the classical invariant theory. The following theorem holds.

Theorem 1. *Let W_d be the following direct sum*

$$W_d = V_2 \oplus V_3 \oplus \cdots \oplus V_d,$$

where V_k is the vector space of binary forms of order k . The algebra of rational simultaneous 2D geometric moment invariants $\mathbb{C}(\eta)_d^{SO(2)}$ is isomorphic to the algebra of joint invariants $\mathbb{C}(W_d)^{SO(2)}$.

Also, we present a minimal generating set of the algebra $\mathbb{C}(W_d)^{SO(2)}$.

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- [1] Hu M. K.: Visual pattern recognition by moment invariants. IRE Trans. Inform. Theory. 8(2), 179–187 (1962)
- [2] Flusser J., Suk T., Zitova B., 2D and 3D Image Analysis by Moments, Wiley, 2017
- [3] Bedratyuk, L.: 2D Geometric Moment Invariants from the Point of View of the Classical Invariant Theory. J Math Imaging Vis (2020)

A. Ya. Beletsky

Effective algorithm for the synthesis of irreducible polynomials

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The known algorithms of IP synthesis usually have no less than a square complexity of calculations. The proposed algorithm of linear complexity based on the so-called *reference stairs (grids)*, the number of steps in which r coincide with the degree n of polynomials f_n to be synthesized. Each step of the staircase put in correspondence with the so-called coordinate vector $CV_r = 10\dots 0$, the number of zeros in which t_r is determined by the ratio $t_r = 2^r - 1$. At each stage, the most straightforward recurrent modular calculations performed, after which the polynomial tested, is unambiguously classified either as non-derived or as a compound.

The concept of singular (exceptional) polynomials introduced, i.e., such IPs whose degrees are: (a) prime numbers, (b) degrees of prime numbers, or (c) the product of two different prime numbers. Empirically, the following have established.

Theorem 1. *A singular polynomial f_n of degree n is irreducible if and only if, when the residue S_n of the coordinate vector CV_r in modulo f_n corresponding to the step n of the reference grid is equal to unity, that is, identity $S_n = 1$ provided (a necessary condition), and under no circumstances $r < n$ the residue S_r can be comparable to the unity (a sufficient condition).*

Theorem 2. *A nonsingular polynomial f_n of degree n is irreducible if and only if, when the residue S_n of the coordinate vector CV_r in modulo f_n equal to unity is a residue corresponding such that identity $S_n = 1$ provided (a necessary condition), and there are no exclusive divisors for the polynomial under test (a sufficient condition).*

The exclusive divisors \tilde{d}_n will include IPs that divide the tested nonsingular polynomial (TNSP) without the remainder.

$$\tilde{d}_n \in \begin{cases} \cup_{i=1}^{k-1} \bar{p}_1^i, & \text{if } n = p_1^k \cdot p_2, k \geq 2; \\ \cup_{i=1}^k \bar{p}_i, & \text{if } n = \cap_{i=1}^k p_i, k > 2; \\ \bar{p}_1 \cup \bar{p}_2 \cup \bar{p}_1 \cdot \bar{p}_2, & \text{if } n = p_1^k \cdot p_2, k \geq 2. \end{cases}$$

A generalized algorithm for the synthesis of irreducible polynomials over a Galois field of arbitrary characteristic p is obtained as the result.

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Hamming spaces and locally matrix algebras

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The (standard) *Hamming space* H_n is the set of all n -tuples $x^n = (x_1, \dots, x_n)$, $x_i \in \{0, 1\}$, $1 \leq i \leq n$, with the distance d_{H_n} , that is defined between two n -tuples as the number of coordinates where they differ.

There are different generalizations of finite metric Hamming space to infinite case, that are constructed as inductive limits of finite Hamming spaces [1], [2]. We define abstract Hamming space, in countable case coincides with known generalizations.

Recall that a Boolean algebra is a commutative algebra over the field $\mathbb{Z}/2\mathbb{Z}$ satisfying the identity $x^2 = x$. A *unital Hamming space* (H, r) is a pair consisting of a Boolean algebra H with 1 and a range function $r : H \rightarrow [0, 1]$ satisfying the following conditions:

- $r(a) = 0$ if and only if $a = 0$,
- $r(a) = 1$ if and only if $a = 1$,
- if $a, b \in H$ and $ab = 0$, then $r(a + b) = r(a) + r(b)$.

In our talk we introduce the operation of tensor product of abstract Hamming spaces. We call a Hamming space locally standard if locally it looks as H_n .

Theorem 1. *Let H be a countable locally standard unital Hamming space. Then $H \cong \otimes_{i=1}^{\infty} H_{p_i}$ in which all H_{p_i} are standard Hamming spaces and all p_i are prime numbers.*

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[2] B. V. Oliynyk and V. I. Sushchanskii, The isometry groups of Hamming spaces of periodic sequences, *Sib. Math. J.* **54** (2013) 124–136.

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On groups with weak Sierpiński subsets

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This is joint work with Y. Cornulier and P. Ślanina.

In a group G , a weak Sierpiński subset is a subset E such that for some $g, h \in G$ and $a \neq b \in E$, we have $gE = E \setminus \{a\}$ and $hE = E \setminus \{b\}$. We study the subgroup generated by g and h , and show that has presentation of the form $G_k = \langle g, h \mid (h^{-1}g)^k \rangle$ or it is free over (g, h) . In addition, in such groups G_k , we characterize all weak Sierpiński subsets.

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On central series of Chernikov p -groups

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We have studied the structure of the upper central series of an arbitrary Chernikov p -group, which is an extension of divisible abelian p -group by the cyclic p -group of the order p . Contrariwise the group structure has been investigated in [1] for a hypercentral p -group with upper central series $Z_0 \subset Z_1 \subset \dots$ continue transfinitely with cyclic factors Z_k/Z_{k-1} of fixed order p^n for some natural n for all finite k .

Definition 1. Let G be a hypercentral group with the central series

$$Z_0 \subset Z_1 \subset \dots \subset Z_\omega \subset \dots \subset G, \quad (1)$$

which is also composition series. We shall call the number of transfinite numbers of the set of member indexes of the series (1) by the transfinite length of this series.

We have proved the next result.

Theorem 1. *Let G be an additive Chernikov p -group with maximal divisible Abelian subgroup M , such that quotient group G/M is the cyclic group of the order p . And let Γ be the integral matrix p -adic representation of the quotient group G/M , induced by the homomorphism $f : g \rightarrow f_g$, $g \in G$, from the group G to the group of automorphisms $\text{Aut } M$, where $f_g(m) = -g + m + g$, $m \in M$. The transfinite length of the composition series of the group G is equal to the number of the irreducible component $g + M \rightarrow 1$ of the representation Γ if G is Abelian group and one more than this number in the case of non-Abelian group G .*

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Extended binary Golay code and principle ideals of group algebras

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The extended binary Golay code is the binary $[24, 12, 8]$ -code, being one of the most interesting self-dual code. It was shown in [1] that this code can be constructed from an principle ideal of the group algebra F_2S_4 over the two-element field F_2 (S_4 denotes, as usual, the symmetric group of degree 4). In [2], the code was constructed from a principle ideal $F_2D_{24}v$ generated by some element v of F_2D_{24} such that $v = \sum_{g \in G} \alpha_g g = \sum_{g \in G} \alpha_g g^{-1} := v^*$ (D_{24} denotes the dihedral group of order 24). In [3], the code was constructed from an F_2Gv with $v = v^*$ for G being one of the following groups of order 24: $C_3 \times D_8$, $C_2 \times A_4$, $(C_6 \times C_2) \rtimes C_2$, and the number of such elements v was calculated. It was also shown that there are no such elements v for other groups of order 24 (note that there are 15 non-isomorphic groups of order 24).

We continue to study the extended binary Golay code from this point of view. We pay particular attention to the most general case (for all mentioned groups G), calculating the number of all elements $v \in F_2G$, for which the extended binary Golay code is constructed from a principle ideal generated by v . In the following table our new results are marked with the symbol +.

G	Number of v , $v = v^*$	Number of all v
	extended binary Golay code constructed from F_2Gv	
D_{24}	768 ⁺	36864 ⁺
S_4	192 ⁺	55296 ⁺
$C_3 \times D_8$	128	12288 ⁺
$C_2 \times A_4$	384	18432 ⁺
$(C_6 \times C_2) \rtimes C_2$	576	27648 ⁺

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On edge-local deformations of quadratic Tits forms of serial Dynkin diagrams

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Let

$$f(z) = f(z_1, \dots, z_n) = \sum_{i=1}^n f_{ii}z_i^2 + \sum_{i<j} f_{ij}z_i z_j$$

be a quadratic form over \mathbb{R} . A deformation of $f(z)$ is (by definition of the first author [1]) a family of quadratic forms parameterized by points of a manifold, one of which corresponds to $f(z)$. The most simple deformations is local ones, i.e. these of the forms

$$f^{(s)}(z, t) = t f_{ss} z_s^2 + \sum_{i \neq s} f_{ii} z_i^2 + \sum_{i < j} f_{ij} z_i z_j \quad \text{with } f_{ss} \neq 0$$

and

$$f^{(p,q)}(z, t) = \sum_{i=1}^n f_{ii} z_i^2 + t f_{pq} z_p z_q + \sum_{(i,j) \neq (p,q)} f_{ij} z_i z_j \quad \text{with } f_{pq} \neq 0,$$

where the parameter t runs \mathbb{R} . The first and second types of local deformations are said to be point-local (or simply local in more early works, in particular in [2]) and edge-local, respectively.

Local deformations were studied in many papers of the first author and his science students. Our new results relate to study edge-local deformations of quadratic Tits forms of serial Dynkin diagrams and are a continuation of those of previous papers (in particular, [3]).

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On Coxeter–Euclidean type of principal posets and generalizations to other posets

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In 2005, the authors classified the posets S with quadratic Tits form $q_S(z)$ being positive (analogs of the Dynkin diagrams) together with the P -critical posets, i.e. the smallest posets T with non-positive $q_T(z)$ (analogs of the extended Dynkin diagrams) [1]. It is well-known (see [2], [3]) that for any poset S with positive Tits form $q_S(z)$ there is a unique (simply-laced) Dynkin diagram DS (called the Coxeter–Dynkin type of S) such that $q_S(z)$ is \mathbb{Z} -equivalent to the quadratic Tits form $q_{DS}(z)$ of the graph DS .

A similar result holds for principal posets, which are a natural generalization of P -critical ones (a poset S is called principal if $q_S(z)$ is non-negative and the symmetric matrix of $q_S(z)$ has corank 1). Namely, for any principal poset S there is a unique (simply faced) extended Dynkin diagram (in other words, Euclidean diagram) DS , called the Coxeter–Euclidean type of S , such that $q_S(z)$ and $q_{DS}(z)$ are \mathbb{Z} -equivalent [4, Proposition 9]. In [5] the author proved that DS can not be a cycle (i.e. \widetilde{A}_s). So we have a solution of Problem 1.6 [6]. Our new results are generalizations of these from [5] to other classes of posets.

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On representations of oversemigroups of the semigroup generated by two mutually annihilating idempotents

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We consider matrix representations of oversemigroups of the semigroup T generated by two mutually annihilating idempotents, $T := (0, b, c) = \langle b, c \rangle: b^2 = b, c^2 = c, bc = 0, cb = 0$; the matrix representations of T are classified in [1].

Denote by (b) , (c) , (bc) , (cb) the defining relations $b^2 = b, c^2 = c, bc = 0, cb = 0$, respectively, and introduce the following semigroups:

$$T^{(b)} := T \setminus (b) := \{0, b, c \mid c^2 = c, bc = 0, cb = 0\};$$

$$T^{(c)} := T \setminus (c) := \{0, b, c \mid b^2 = b, bc = 0, cb = 0\};$$

$$T^{(bc)} := T \setminus (bc) := \{0, b, c \mid b^2 = b, c^2 = c, cb = 0\};$$

$$T^{(cb)} := T \setminus (cb) := \{0, b, c \mid b^2 = b, c^2 = c, bc = 0\}.$$

Put

$$T^{(x,y)} := T \setminus \{(x), (y)\} \text{ for } x, y \in \{(b), (c), (bc), (cb)\}, x \neq y;$$

$$T^{(x,y,z)} := T \setminus \{(x), (y), (z)\} \text{ for } x, y, z \in \{(b), (c), (bc), (cb)\}, x \neq y,$$

$x \neq z, y \neq z$.

Theorem. *For an arbitrary field K ,*

- 1) $T^{(x)}$ is a semigroup of finite representation type for $x \in \{(bc), (cb)\}$;
- 2) $T^{(x)}$ is a tame semigroup of infinite representation type for $x \in \{(b), (c)\}$;
- 3) $T^{(x,y)}$ is a tame semigroup of infinite representation type for $x, y \in \{(b), (c)\}$ or $x, y \in \{(bc), (cb)\}$;
- 4) $T^{(x,y)}$ is a wild semigroup for $x \in \{(b), (c)\}, y \in \{(bc), (cb)\}$ or $x \in \{(bc), (cb)\}, y \in \{(b), (c)\}$;
- 5) $T^{(x,y,z)}$ is a wild semigroup for all $x, y, z \in \{(b), (c), (bc), (cb)\}$.

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On 2-hereditary reducibility of some canonically cyclic monomial matrices over commutative local rings

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Let K be a commutative ring with identity. A canonically cyclic (monomial) matrix

$$A = \begin{pmatrix} 0 & \dots & 0 & a_n \\ a_1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & a_{n-1} & 0 \end{pmatrix}$$

over K (with defining sequence (a_1, a_2, \dots, a_n)) is called 2-hereditary reducible over K if the matrix A similar to

$$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix},$$

where matrices A_{11} and A_{22} are also canonically cyclic [1].

If K is a commutative local ring with Jacobson radical $R \neq 0$ and $t \in R$ then a canonically cyclic matrix A with defining sequence $(\varepsilon_1 t^{s_1}, \varepsilon_2 t^{s_2}, \dots, \varepsilon_n t^{s_n})$, where $\varepsilon_i \in K^*$ ($i = 1, 2, \dots, n$), is called by us canonically $(t, *)$ -cyclic.

Let K be a commutative local ring with Jacobson radical $R \neq 0$ and let t be a non-zero element of R such that $t^m = 0$, $t^{m-1} \neq 0$.

Theorem 1. *Any canonically $(t, *)$ -cyclic monomial matrix A is 2-hereditary reducible over K if the defining sequence of A contains subsequences (t^i, t^{p+q}, t^j) and (1) , where $i + q \geq m$, $j + p \geq m$.*

Theorem 2. *Any canonically $(t, *)$ -cyclic monomial matrix A is 2-hereditary reducible over K if the defining sequence of A contains subsequences $(t^i, t^u, t^j, 1, t^v)$ and $(1, 1, 1)$, where $i + u \geq m$, $j + v \geq m$.*

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The structure of elements of the Zelisko group

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A ring R has stable range 1.5 if for each $a, b \in R$ and $c \in R \setminus \{0\}$ such that $aR + bR + cR = R$ there exists $r \in R$ such that $(a + br)R + cR = R$.

Each commutative Bézout domain R of stable range 1.5 is an elementary divisor ring, i.e. for each $A \in R^{m \times n}$ there exist invertible matrices $P \in R^{m \times m}$ and $Q \in R^{n \times n}$ such that

$$PAQ = \text{diag}(\varphi_1, \dots, \varphi_k, 0, \dots, 0) = \Phi,$$

where $\varphi_i \mid \varphi_{i+1}$ for $i = 1, \dots, n - 1$. The matrix Φ is called the Smith normal form and P, Q are left and right transforming matrices of A .

To a matrix $\Phi \in R^{n \times n}$ we correspond a subgroup $\mathbf{G}_\Phi \leq \text{GL}_n(R)$ which is called the Zelisko group of Φ and it is definite as:

$$\mathbf{G}_\Phi = \{H \in \text{GL}_n(R) \mid \exists S \in \text{GL}_n(R) : H\Phi = \Phi S\}.$$

For each $m \in R$ we define the factor ring $R_m = R/mR$.

In our talk we give descriptions of elements of the Zelisko group \mathbf{G}_Φ of a nonsingular matrix $\Phi = \text{diag}(\varphi_1, \dots, \varphi_n) \in R_m^{n \times n}$, where $n \geq 2$.

Theorem 1. *Among solutions of a solvable equation $ax = b$ in R_m there exists a solution which divides all other.*

The solutions of a solvable equation $ax = b$ which divides all other are called generating solutions.

Theorem 2. *The generating solutions of a solvable equation $ax = b$ are pairwise associable.*

The set of generating elements of each solvable equation $ax = b$ contains a minimal element with respect to an ordering relation \preceq . This minimal element we denoted by $\langle \frac{b}{a} \rangle$.

Theorem 3. *Let $\Phi = \text{diag}(\varphi_1, \dots, \varphi_n) \in R_m^{n \times n}$ be nonsingular. Then the Zelisko group \mathbf{G}_Φ consists of all invertible matrices of the form*

$$\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1, n-1} & h_{1n} \\ \langle \frac{\varphi_2}{\varphi_1} \rangle h_{21} & h_{22} & \cdots & h_{2, n-1} & h_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \langle \frac{\varphi_n}{\varphi_1} \rangle h_{n1} & \langle \frac{\varphi_n}{\varphi_2} \rangle h_{n2} & \cdots & \langle \frac{\varphi_n}{\varphi_{n-1}} \rangle h_{n, n-1} & h_{nn} \end{bmatrix}, \quad (h_{ij} \in R_m).$$

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Krawtchouk matrices and Zavadskij's calculus as applications of the theory of representation of equipped posets

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The theory of representation of equipped posets was introduced and developed by A.G. Zavadskij and his students in Kiev and Colombia in the 1990's and 2000's, such theory can be viewed as a way to address the biquadratic homogeneous problem. The description of this problem deal with generalizations to the Kronecker problem for instance, the semilinear Kronecker problem solved by Djoković as well as the more general pseudo-linear bundle case introduced by Sergeichuk who investigated systems of linear differential equations of the form $A(t)y' + B(t)y = f(t)$ by determining the canonical form of the pairs $(A(t), B(t))$ [2–4].

In order to describe solutions of the biquadratic homogeneous problem, Zavadskij rediscovered the Krawtchouk matrices giving some of its combinatorial properties. We recall that Krawtchouk matrices have been a tool to solve problems in different areas, e.g., quantum computing, probability, Lie algebras, representation theory, statistics, differential equations, combinatorics, etc [1].

In this talk, we describe some topics regarding the theory of representation of equipped posets (in particular categorical properties of some algorithms of differentiation) its relationship with the biquadratic homogeneous problem and the Zavadskij calculus which is a tool to solve systems of non-linear differential equations of the form $y'(t) + y^2(t)A = B$, where A and B are square matrices of the same size. We will see that Krawtchouk matrices allow to obtain explicit solutions of these kind of systems of differential equations.

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Metabelian nilpotent Lie algebras of derivations

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Let \mathbb{K} be an algebraically closed field of characteristic zero, \mathbb{A} an integral domain over \mathbb{K} , R the quotient field $\text{Frac}A$, and $\text{Der}_{\mathbb{K}}A$ the Lie algebra of all \mathbb{K} -derivations on \mathbb{A} . Denote by $W(A)$ the Lie subalgebra $R\text{Der}_{\mathbb{K}}A \subseteq \text{Der}_{\mathbb{K}}R$ and by $u_n(\mathbb{K}) = \mathbb{K}[x_2, \dots, x_n] \frac{\partial}{\partial x_1} + \dots + \mathbb{K} \frac{\partial}{\partial x_n}$ the triangular Lie algebra.

For a Lie subalgebra $L \subseteq W(A)$ define the rank of L over R as $\text{rank}_R L = \dim_R RL$ (note that RL is a vector space over R , but not a Lie algebra). If I is an ideal of a Lie algebra $L \subseteq W(A)$, then $RI \cap L$ is an ideal of L . We consider nilpotent Lie subalgebras $L \subseteq W(A)$ of rank n such that the quotient algebra $L/(RZ \cap L)$ is abelian, where $Z = Z(L)$ is the center of L (note that these Lie algebras are metabelian). We show that the Lie algebra FL (as a Lie algebra over F) can be isomorphically embedded in the triangular Lie algebra $u_n(F)$, where $F = R^L$ is the field of constants. This result generalizes the main results of the papers [4], [5]. The triangular Lie algebra was studied in [1], [2], some properties of nilpotent Lie subalgebras of $W(A)$ were given in [3].

Theorem 1. *Let $L \subset W(A)$ be a nilpotent Lie subalgebra of finite rank n over R with the field of constants $F = R^L$. If $L/(RZ \cap L)$ is abelian, then the Lie algebra FL can be isomorphically embedded in the triangular Lie algebra $u_n(F)$ (as an abstract Lie algebra).*

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**Calabi-Yau algebras and noncommutative symplectic
geometry**

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The notion of Calabi-Yau algebras was introduced by Ginzburg in 2007 and has been intensively studied in the past decade. In this talk, we show that there is a noncommutative, shifted symplectic structure on Koszul Calabi-Yau algebras, which also introduces a shifted symplectic structure on their derived character varieties (a notion introduced by Berest et al). We will also discuss the quantization of the symplectic structure, and show that it is induced from (a version of) the quantization on the Calabi-Yau algebras.

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On the structure of Leibniz algebras all of whose not ideal subalgebras are core-free

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An algebra L over a field F is said to be a Leibniz algebra if it satisfies the Leibniz identity: $[[a, b], c] = [a, [b, c]] - [b, [a, c]]$ for all $a, b, c \in L$. A subalgebra S of a Leibniz algebra L is called a *core-free*, if S does not include a non-zero ideal. Let L be a Leibniz algebra. The intersection of all non-zero ideals $Mon(L)$ of L is called the *monolith* of a Leibniz algebra L . If $Mon(L) \neq \langle 0 \rangle$, then the Leibniz algebra L is called *monolithic*. It was obtained a description of a structure of a Leibniz algebras whose subalgebras are ideals or core-free.

Theorem 1. *Let L be a non-monolithic Leibniz algebra. If every subalgebra of L , which is not an ideal, is core-free, then every subalgebra of L is an ideal.*

Theorem 2. *Let L be a Leibniz algebra. Suppose that L includes a subalgebra, which is not an ideal, and every subalgebra of L , which is not an ideal, is core-free. If the center of L is non-zero, then L satisfies the following conditions:*

- (i) L is monolithic, and $Mon(L) = \zeta(L) = \gamma_3(L)$, in particular, $\dim_F(\zeta(L)) = 1$;
- (ii) $\gamma_2(L) = [L, L] \leq \zeta_2(L)$ and $\gamma_2(L)$ has dimension 2;
- (iii) every subalgebra of L , which is not an ideal, is Abelian;
- (iv) every subalgebra of $L/\zeta(L)$ is an ideal.

Conversely, if L is a Leibniz algebra satisfying the above conditions, then every subalgebra of L either is core-free or an ideal.

Theorem 3. *Let L be a monolithic Leibniz algebra whose center is zero. Suppose that every subalgebra of L , which is not an ideal, is core-free. If L is not a Lie algebra, then the following conditions hold:*

- (i) $Mon(L)$ is a minimal ideal of L ;
- (ii) $Mon(L)$ is a maximal Abelian ideal of L ;
- (iii) $L = Mon(L) \oplus A$ for some Abelian subalgebra A ;
- (iv) $Ann_L(Mon(L)) = Ann_L^{left}(Mon(L)) = Mon(L)$.

Conversely, if L is a Leibniz algebra satisfying the above conditions, then every subalgebra of L either is core-free or an ideal.

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Adequate properties in duo rings

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All rings considered will be associative and have nonzero identity.

An element a of the Bezout ring R is called a right adequate element if for any element $b \in R$ there exist elements $r, s \in R$ so that: 1) $a = rs$; 2) $bR + rR = R$; 3) $\forall s' \in R, sR \subset s'R \neq R \Rightarrow bR + s'R \neq R$ [1].

A ring R is said to be a *clean ring* if for any element $x \in R$ there exists an invertible element $u \in R$ and an idempotent $e \in R$ such that $x = u + e$ [2].

A ring R is called an exchange ring if, for every $x \in R$, there is an idempotent $e \in R$ such that $e \in xR$ and $(1 - e) \in (1 - x)R$. Von Neumann regular rings are examples of exchange rings.

A ring R is called *duo-ring* if every right and left ideal in R is an ideal.

A ring R is said to be a *ring of an idempotent stable range 1* if for any $a, b \in R$ such that $Ra + Rb = R$, there exists an idempotent $e \in R$ such that $a + eb$ is a unit of R [3].

Theorem 1. *Let a be a right adequate element of a duo-ring R . Then zero is right adequate element of the factor-ring R/aR .*

Theorem 2. *Let R be a duo ring in which zero is right adequate element. Then R is a ring of an idempotent stable range 1.*

Theorem 3. *Let R be a duo ring in which zero is right adequate element. Then the following statements hold:*

- 1) R is a clean ring.
- 2) R is an exchange ring.
- 3) R is a ring of an idempotent stable range 1.

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Grothendieck group of a locally finite category

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A *locally finite category* over a Dedekind domain R is a category \mathcal{A} such that all sets of morphisms $\mathcal{A}(A, B)$ are finitely generated R -modules and the multiplication of morphisms is R -bilinear. We denote by Q the field of fractions of R , by $\hat{R}_{\mathfrak{p}}$ the \mathfrak{p} -adic completion of the local ring $R_{\mathfrak{p}}$, by $\hat{Q}_{\mathfrak{p}}$ the field of fractions of $\hat{R}_{\mathfrak{p}}$ and consider the categories $Q\mathcal{A}$, $\mathcal{A}_{\mathfrak{p}}$, $\hat{\mathcal{A}}_{\mathfrak{p}}$ and $Q\hat{\mathcal{A}}_{\mathfrak{p}}$ obtained from \mathcal{A} by extending scalars from R , respectively, to Q , $R_{\mathfrak{p}}$, $\hat{R}_{\mathfrak{p}}$ and $\hat{Q}_{\mathfrak{p}}$. We calculate the Grothendieck group $K_0(\mathcal{A})$ under the next restriction, which holds true in most situations.

Definition 1. We say that \mathcal{A} *satisfies the Max-condition* if for every indecomposable object $V \in Q\mathcal{A}$ there is an object $S(V) \in \mathcal{A}$ such that $QS \simeq V$ and $\Delta(V) = \text{End}_{\mathcal{A}} S(V) / \text{nil End}_{\mathcal{A}} S(V)$ is a maximal order in the semisimple Q -algebra $\text{End}_{Q\mathcal{A}} V / \text{nil End}_{Q\mathcal{A}} V$.

We denote by $G(A)$ the *genus* of an object $A \in \mathcal{A}$, that is the set of objects A' such that $A'_{\mathfrak{p}} \simeq A_{\mathfrak{p}}$ for all prime \mathfrak{p} , and consider the Grothendieck group of genera $K_0(G\mathcal{A})$.

Theorem 1. *If a locally finite category \mathcal{A} satisfies the Max-condition, $K_0(\mathcal{A}) = \text{Cl}(\mathcal{A}) \oplus K_0(G\mathcal{A})$, where $K_0(G\mathcal{A})$ is a free abelian group which is the glueing of free groups $K_0(\mathcal{A}_{\mathfrak{p}})$ along the common quotient $K_0(Q\mathcal{A})$ and $\text{Cl}(\mathcal{A}) = \bigoplus_V \text{Cl}(\Delta(V))$, where V runs through the isomorphism classes of indecomposable objects from $Q\mathcal{A}$ and $\text{Cl}(\Delta(V))$ denotes the group of the ideal classes of the maximal order $\Delta(V)$ [2].*

As an immediate corollary, we obtain a new proof of the Freyd theorem on the Grothendieck group of the *stable homotopy category of polyhedra* SW , see [1].

Theorem 2 (Freyd). *$K_0(SW)$ is a free abelian group with a basis consisting of genera of indecomposable p -primary polyhedra.*

Here a *p -primary polyhedron* A is such that A_q is isomorphic to a bouquet of spheres for all primes q except exactly one prime p .

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On some applications of classification of low-dimensional Lie algebras

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We plan to present a short review of some applications of classification of low-dimensional Lie algebras. The details can be found in:

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Localization functors and Gelfand-Tsetlin modules

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Localization functors, introduced by V. Deodhar, are important tools in the representation theory. They were used successfully in various classification problems. We study localization functors assigned to an arbitrary non-simple root of a Lie algebra on the category \mathcal{O} . This allows to construct explicitly a large new class of simple modules together with their geometric realization. The talk is based on joint results with Libor Křížka.

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The automorphism group of a variant of a power set

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Let (L, \leq) be a lower semilattice, $a \in L$ is an arbitrary element. A lower conus of a we will denote $\Delta_a = \{x \in L \mid x \wedge a = x\}$.

Proposition 1. *Let $(L, *_a)$ be a variant, $a \in L$. Then a set of generating elements of the variant contains $L \setminus \Delta_a$.*

Let $\mathfrak{B}(M)$ be a power set of $M = \{1, 2, \dots, n\}$. If the set $A \subset N$ then by $\mathfrak{B}(A)$ we denote a power set of A .

Proposition 2. *Let $(\mathfrak{B}(M), *_A)$ be a variant generated by the set A . Then $\mathfrak{B}(M) \setminus \mathfrak{B}(A)$ is a set of generating elements.*

Theorem 3. *Let $\text{Aut}(\mathfrak{B}(M), *_A)$ be an automorphism group of the variant $(\mathfrak{B}(M), *_A)$, $|M| = n$ and $|A| = k$. Then $\text{Aut}(\mathfrak{B}(M), *_A)$ is isomorphic to a wreath product $\widehat{S}_k \wr S_{2^{n-k}-1}$ of symmetric groups \widehat{S}_k and $S_{2^{n-k}-1}$, and \widehat{S}_k is an induced action of a symmetric group S_k on the subsets of the set $\{1, 2, \dots, k\}$.*

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Bezout rings of neat range 1

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All rings considered will be commutative and have identity.

We introduce the necessary definitions and facts.

By a Bezout ring we mean a ring in which all finitely generated ideals are principal. An n by m matrix $A = (a_{ij})$ is said to be diagonal if $a_{ij} = 0$ for all $i \neq j$. We say that a matrix A of the dimension n by m admits a diagonal reduction if there exist invertible matrices $P \in GL_n(R)$, $Q \in GL_m(R)$ such that PAQ is a diagonal matrix. We say that two matrices A and B over a ring R are equivalent if there exist invertible matrices P, Q such that $B = PAQ$. Following Kaplansky [1], we say that if every matrix over R is equivalent to a diagonal matrix (d_{ii}) with the property that every (d_{ii}) is a divisor of $d_{i+1, i+1}$, then R is an elementary divisor ring. An element $a \in R$ is called clean if a can be written as the sum of a unit and an idempotent. If each element of R is clean, then we say R is a clean ring [2]. A ring R is said to have stable range 2 if for all $a, b, c \in R$ such that $aR + bR + cR = R$, there exists $x, y \in R$ such that $(a + cx)R + (b + cy)R = R$.

Definition 1. An element $a \in R$ is called a neat element if the ring R/aR is a clean ring.

Definition 2. A ring R is said to be a ring of neat range 1 if for any elements $a, b \in R$ such that $aR + bR = R$ and for any nonzero element $c \in R$ there exist such elements $u, v, t \in R$ that $a + bt = uv$, where $uR + cR = R$, $vR + (1 - c)R = R$, and $uR + vR = R$.

Theorem 1. *Let R be a Bezout ring of neat range 1. Then R is a ring of stable range 2.*

Theorem 2. *Let R be a Bezout ring of neat range 1. Then R is an elementary divisor ring.*

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Joint research (1977-1984 years) of the department of information languages and systems of IK AN USSR and the department of algebra and mathematical logic of KSU on the algorithmic and semantic aspects of algebraic problems of combinatorics, and modern prospects

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During 1977-1984 years above mentioned in the abstract title research have be implemented. From the side of KSU, the research was led by L.A. Kaluzhnin, and from the side of the Cybernetics Institute of the Academy of Sciences of the Ukrainian SSR, A.A. Stogniy. The conducted studies are reflected in the publications of participants in the collections of works:

Computations in algebra and combinatorial analysis. Kiev: Inst. of Cyb. Ac. Sci. Ukr. SSR, 1978.,

Computations in algebra, combinatorics and number theory. Kiev: Inst. of Cyb. Ac.Ukr.SSR.1980.,

as well as in publications

Kalughnin L.A., Stogny A.A., Glazunov N.M. Sushchansky V.I. Problems Concerned with Development of Algebraic Computer Aided Research. *Cybernetics (Kibernetika, Kiev)*, 1983, no. 2, pp.1-10.

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and in others.

In the process of these studies, the author developed and proposed a conceptual model for the integration of knowledge, algorithms, and software in group theory, in the theory of algebraic groups, and in the theory of formal groups.

Then I have extended this model on semantic ontologies.

In the present communication I plan to present the current state of the model.

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R. Golovko

On holomorphic Cthulhu

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In this talk we present the definition of Floer theory for Lagrangian cobordisms (Cthulhu homology) and its applications. This is joint work with Baptiste Chantraine, Georgios Dimitroglou Rizell and Paolo Ghiggini.

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Normal subgroups in the group of column-finite infinite matrices

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The classical result, due to Jordan, Burnside, Dickson, says that every normal subgroup of $GL(n, K)$ (K - a field, $n \geq 3$) which is not contained in the center, contains $SL(n, K)$. A. Rosenberg gave description of normal subgroups of $GL(V)$, where V is a vector space of any infinite cardinality dimension. However, when he considers subgroups of the group of linear transformations g such that $g - id_V$ has finite dimensional ranges the proof is not complete. We fill this gap for countably dimensional V giving description of the lattice of normal subgroups in the group of infinite column-finite matrices indexed by positive integers over any field.

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On generalized binary quasigroup functional equations of the type (5; 2; 0)

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Let Q be a set. A mapping $f : Q^2 \rightarrow Q$ is called a *binary invertible function*, if there exist functions f_1 and f_2 such that the identities $f(f_1(x, y), y) = x$, $f_1(f(x, y), y) = x$, $f(x, f_2(x, y)) = y$, $f_2(x, f(x, y)) = y$ hold. A *generalized binary quasigroup functional equation of the type (5; 2; 0)* means that all functional variables are pairwise different and take on their values in the set of all binary invertible functions defined on a set Q and it has two individual variables with 5 and 2 appearances. Non-triviality implies that each individual variable appears at least twice. A *length* of a functional equation is a number of all occurrences of all functional variables in the equation including their repetitions. Two equations are said to be *parastrophically-primarily equivalent* [1], if one can be got from the other by applying the equalities given above.

Theorem 1. *There exist at least 11 non-trivial generalized binary quasigroup functional equations of the length 5 of the type (5, 2, 0) up to a parastrophically primary equivalence:*

$$\begin{aligned}
 F_1(x; F_2(y; y)) &= F_3(F_4(x; x); F_5(x; x)), \\
 F_1(x; F_2(x; x)) &= F_3(F_4(x; x); F_5(y; y)), \\
 F_1(x; F_2(y; y)) &= F_3(x; F_4(x; F_5(x; x))), \\
 F_1(x; F_2(x; y)) &= F_3(F_4(x; x); F_5(x; y)), \\
 F_1(x; F_2(x; x)) &= F_3(F_4(x; y); F_5(x; y)), \\
 F_1(y; F_2(x; y)) &= F_3(F_4(x; x); F_5(x; x)), \\
 F_1(y; F_2(x; x)) &= F_3(F_4(x; x); F_5(x; y)), \\
 F_1(x; F_2(x; y)) &= F_3(x; F_4(x; F_5(x; y))), \\
 F_1(y; F_2(x; x)) &= F_3(y; F_4(x; F_5(x; x))), \\
 F_1(y; F_2(x; x)) &= F_3(x; F_4(y; F_5(x; x))), \\
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 \end{aligned}$$

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On free n -trinilpotent triods

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Recall that a trioid [1] is a nonempty set T equipped with three binary associative operations \dashv , \vdash , and \perp satisfying the following eight axioms:

$$\begin{aligned} (x \dashv y) \dashv z &= x \dashv (y \vdash z), & (x \vdash y) \dashv z &= x \vdash (y \dashv z), \\ (x \dashv y) \vdash z &= x \vdash (y \vdash z), & (x \dashv y) \dashv z &= x \dashv (y \perp z), \\ (x \perp y) \dashv z &= x \perp (y \dashv z), & (x \dashv y) \perp z &= x \perp (y \vdash z), \\ (x \vdash y) \perp z &= x \vdash (y \perp z), & (x \perp y) \vdash z &= x \vdash (y \vdash z) \end{aligned}$$

for all $x, y, z \in T$.

An element 0 of a trioid $(T, \dashv, \vdash, \perp)$ is called zero [3], if $x * 0 = 0 = 0 * x$ for all $x \in T$ and $*$ $\in \{\dashv, \vdash, \perp\}$. As usual, \mathbb{N} denotes the set of all positive integers. A semigroup S with zero 0 is called nilpotent, if $S^{n+1} = 0$ for some $n \in \mathbb{N}$. The least such n is called the nilpotency index of S . For $k \in \mathbb{N}$ a nilpotent semigroup of nilpotency index $\leq k$ is said to be k -nilpotent. A trioid $(T, \dashv, \vdash, \perp)$ with zero will be called trinilpotent, if (T, \dashv) , (T, \vdash) and (T, \perp) are nilpotent semigroups. A trinilpotent trioid $(T, \dashv, \vdash, \perp)$ will be called n -trinilpotent, if (T, \dashv) , (T, \vdash) and (T, \perp) are n -nilpotent semigroups.

The notion of a trinilpotent trioid is an analog of notions of a dinilpotent dimonoid [2] and a dinilpotent doppelsemigroup [4]. It is clear that operations of any 1-trinilpotent trioid coincide and it is a zero semigroup. The class of all n -trinilpotent trioids forms a subvariety of the variety of trioids. A trioid which is free in the variety of n -trinilpotent trioids will be called a free n -trinilpotent trioid.

We solve the problem of constructing a free n -trinilpotent trioid. We also consider separately free n -trinilpotent trioids of rank 1.

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The relation between the diagonal entries and the eigenvalues of a matrix

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The well-known Brauer's theorem describes one of the ways to adjust the eigenvalue of a matrix.

The application of this theorem played a key role in early website ranking methods and the emergence of the Google matrix.

In some applications, we would like to adjust an eigenvalue of a matrix by adding a suitable identity matrix I multiplied by a scalar (real or complex).

The following theorems describe the relation between the diagonal entries and the eigenvalues of a matrix.

Theorem 1. *Let $A \in M_n(\mathbb{C})$ and let $\lambda_1, \lambda_2, \dots, \lambda_n$ be its eigenvalues including multiplicities.*

Let $k \geq 2$ and let $A_1, A_2, \dots, A_k \in M_n(\mathbb{C})$ such that $A_l = A + lI$, $l = 1, \dots, k$. Then

- 1) $A_i A_j = A_j A_i$ for all $i, j = 1, \dots, k$;*
- 2) the scalars $\lambda_1 + l, \lambda_2 + l, \dots, \lambda_n + l$ are eigenvalues of A_l , $l = 1, \dots, k$;*
- 3) the eigenvectors of A_l , $l = 1, \dots, k$ and A coincide.*

Theorem 2. *Let $A \in M_n(\mathbb{C})$ and let $k \geq 2$, $A_l = A + lI$, $l = 1, \dots, k$. There is some ordering $\lambda^i_1, \lambda^i_2, \dots, \lambda^i_n$ of the eigenvalues of A_i and some ordering $\mu^j_1, \mu^j_2, \dots, \mu^j_n$ of the eigenvalues of A_j such that $\lambda^i_1 + \mu^j_1, \lambda^i_2 + \mu^j_2, \dots, \lambda^i_n + \mu^j_n$ are the eigenvalues of $A_i + A_j$ for all $i, j = 1, \dots, k$ and $\lambda^i_1 \mu^j_1, \lambda^i_2 \mu^j_2, \dots, \lambda^i_n \mu^j_n$ are the eigenvalues of $A_i A_j$ for all $i, j = 1, \dots, k$.*

In this case, the eigenvectors that correspond to the corresponding eigenvalues of the matrices $A_i + A_j$, $A_i A_j$ for all $i, j = 1, \dots, k$ and A coincide.

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***F*-inverse monoids in enriched signature**

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Every F -inverse monoid can be equipped with a second unary operation $a \mapsto a^m$. Here a^m denotes the greatest element in the σ -class of a . In this enriched signature, the class of all F -inverse monoids forms a variety of algebraic structures. We describe universal objects in several classes of F -inverse monoids, in particular free F -inverse monoids. More precisely, for every X -generated group G we describe the initial object in the category of all X -generated F -inverse monoids F for which $F/\sigma = G$.

This is a joint work with K. Auinger (Vienna) and M. B. Szendrei (Szeged).

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Groups containing only normal, contranormal and core-free subgroups

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Let G be a group. The following two normal subgroups are associated with any subgroup H of G : H^G , the *normal closure* of H in G , and $\mathbf{Core}_G(H)$, the *normal core* of H in G . We have $H^G = \langle H^x \mid x \in G \rangle$ and $\mathbf{Core}_G(H) = \bigcap_{x \in G} H^x$.

A subgroup H is normal in G iff $\mathbf{Core}_G(H) = H$. In this sense, the subgroups H , for which $\mathbf{Core}_G(H) = \langle 1 \rangle$, are the complete opposite to the normal subgroups. A subgroup H of G is called *core-free* in G if $\mathbf{Core}_G(H) = \langle 1 \rangle$. A subgroup H is normal in G iff $H^G = H$. In this sense, the subgroups H , for which $H^G = G$, are the complete opposites to the normal subgroups. A subgroup H of G is called *contranormal* in G if $H^G = G$ [2].

For each subgroup H of a group G we have the following two extreme and opposite situations: $H^G = H$ or $H^G = G$, and, respectively, $\mathbf{Core}_G(H) = H$ or $\mathbf{Core}_G(H) = \langle 1 \rangle$. Groups, whose subgroups are either normal or core-free, have been studied in [1]. The study of groups, whose subgroups are either normal or contranormal was initiated in [3]. We have substantially supplemented the results of the last paper.

Theorem A. *Let G be a group, whose non-normal subgroups are contranormal. If G is not soluble, then G is simple or quasisimple.*

Also we obtained a detailed description of soluble groups whose non-normal subgroups are contranormal (8 types of groups). Moreover, we obtained a very detailed description of some soluble periodic groups whose non-normal subgroups are either contranormal or core-free (24 types of groups). Separately, we can distinguish the following

Theorem B. *Let G be a group, whose non-normal subgroups are either contranormal or core-free. If G is locally soluble, then G is a soluble group.*

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On Leibniz algebras whose subalgebras are left ideals

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Let L be an algebra over a field F with the binary operations $+$ and $[\cdot, \cdot]$. Then L is called a *Leibniz algebra* (more precisely a *left Leibniz algebra*), if it satisfies the (left) Leibniz identity:

$$[[a, [b, c]] = [[a, b], c] + [b, [a, c]] \text{ for all } a, b, c \in L.$$

Note that the Lie algebras are the partial case of Leibniz algebras. Thus, Lie algebras can be characterized as anticommutative Leibniz algebras. The significant difference between Lie algebras and Leibniz algebras, which are distinguished by the same restriction, can be traced in [1–3]. In particular, such a situation takes place for the Leibniz algebras whose subalgebras are ideals. Again, in the case of Lie algebras, we arrive at the Abelian algebras, while in the case of Leibniz algebras, the following interesting type of algebras arises here.

Theorem. *Let L be a Leibniz algebra over a field F , and let all subalgebras of L be left ideals. Suppose that L is cyclic and not abelian.*

- (i) *If L is nilpotent, then $L = Fa \oplus Fb$, where $b = [a, a]$, $[b, a] = [a, b] = 0$.*
- (ii) *If L is not nilpotent, then $L = \langle a \rangle = Fd \oplus Fb$, where $b = [a, a]$, $[d, d] = [a, d] = [b, b] = [b, a] = [b, d] = 0$, $[d, a] = b$ and $[a, b] = [d, b] = \lambda b$, where λ is a non-zero element of field F .*
- (iii) *If L is not nilpotent, $L = Fd \oplus Fb$, where $[d, d] = 0 = [b, b]$, then L is cyclic.*

Conversely, if L is a Leibniz algebra of the above types (i) or (ii), then every subalgebra of L is a left ideal.

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Minimal generating sets in wreath powers

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Let (T, X) be a finite transformation semigroup or permutation group. The infinite wreath power $W^\infty(T, X)$ of (T, X) is the semigroup of transformations or group of permutations of the free monoid X^* defined by initial automata over X such that in every their state only transformations or permutations from (T, X) appear. The definition of the finite state wreath power $FW^\infty(T, X)$ of (T, X) include additional requirement for automata to be finite. In [1] it is found sufficient conditions under which $FW^\infty(T, X)$ posses no minimal generating sets. From the other hand, in [2] it is proved that infinite and finite state wreath powers of finite symmetric groups naturally acting on underlying sets contain minimal generating sets.

Let p be a prime.

Theorem 1. *Infinite wreath power $W^\infty(\mathbb{Z}_p, \mathbb{Z}_p)$ of the regular cyclic group of order p contains a minimal generating set.*

Theorem 2. *Finite state wreath power $FW^\infty(\mathbb{Z}_p, \mathbb{Z}_p)$ of the regular cyclic group of order p contains a minimal generating set.*

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Quantum Markov chains, a unification approach

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In the present paper we study a unified approach for Quantum Markov Chains (QMC). A new quantum Markov property that generalizes the old one, is discussed. We introduce Markov states and chains on general local algebras, possessing a generic algebraic property. We stress that this kind of algebras includes both Boson and Fermi algebras. Our main results concern two reconstruction theorems for quantum Markov chains and for quantum Markov states. Namely, we illustrate the results through examples.

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Locally soluble groups with the restrictions on the generalized norms

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In group theory one of focuses is on the findings related to the study of groups, in which the subgroups (or the system of subgroups) with some theoretical group property satisfy given restrictions. In some cases the existence of one characteristic subgroup with a certain property can be the determining factor for the structure of the group. Different Σ -norms of a group are the subgroups of such a type.

The author continues the investigation of connections between given Σ -norms of groups and studies the properties of locally soluble groups in which the norm of decomposable subgroups and the norm of Abelian non-cyclic subgroups are nonidentity and have the identity intersection.

The intersection N_G^d of normalizers of all decomposable subgroups of a group G is called the norm of decomposable subgroups of G . Accordingly, the intersection N_G^A of normalizers of all non-cyclic Abelian subgroups of G (provided that the system of these subgroups is non-empty) is called the norm of non-cyclic Abelian subgroups of G [1].

Theorem 1. *If a locally soluble group G contains an Abelian non-cyclic subgroup, the norms N_G^d and N_G^A are nonidentity and $N_G^d \cap N_G^A = E$, then G is a non-periodic group.*

Theorem 2. *If a non-periodic locally soluble group G has an Abelian non-cyclic subgroup, the norm N_G^A is non-Dedekind, the norm N_G^d is nonidentity and $N_G^d \cap N_G^A = E$, then the following conditions take place:*

- 1) *the norm $N_G^d = \langle c \rangle$ is a cyclic group of a prime odd order p ;*
- 2) *the norm N_G^A is a group of the type $N_G^A = A \rtimes \langle b \rangle$, where A is a group isomorphic to an additive group of p -adic fractions (p is prime, $(p, 2) = 1$), $|b| = 2$ and $b^{-1}ab = a^{-1}$ for any $a \in A$;*
- 3) *every infinite cyclic subgroup has a nonidentity intersection with the norm N_G^A ;*
- 4) *a group G does not contain a free Abelian subgroups of rank 2 and a finite non-cyclic Abelian subgroups;*
- 5) *$Z(G) = E$ and the factor-group G/N_G^A is periodic.*

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On finite 2-groups with the non-Dedekind metacyclic norm of abelian non-cyclic subgroups

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One of productive directions in group theory is the study of influence of some characteristic subgroups on the structure of the whole group. Such characteristic subgroups include different Σ -norms of a group. Recall, that the intersection of the normalizers of all subgroups of a system Σ , provided that Σ contains all subgroups of a group with some property is called a Σ -norm of a group G .

The authors continue the investigation of finite 2-groups with the non-Dedekind norm of Abelian non-cyclic subgroups, initiated in [1]-[2]. The structure of finite 2-groups with the cyclic center and the metacyclic non-Dedekind norm N_G^A of Abelian non-cyclic subgroups is described in the following theorem.

Theorem 1. *An arbitrary finite 2-group with the cyclic center and the non-Dedekind metacyclic norm N_G^A of Abelian non-cyclic subgroups is a group of one of the following types:*

- 1) $G = \langle a \rangle \langle b \rangle, |a| = 2^n, n > 2, |b| = 8, b^4 = a^{2^{n-1}}, b^{-1}ab = a^{-1}, N_G^A = G;$
- 2) $G = \langle a \rangle \rtimes \langle b \rangle, |a| = 2^n, |b| = 2^m, n \geq 2, m \geq 1, [a, b] = a^{2^{n-1}}, N_G^A = G;$
- 3) $G = \langle a \rangle \rtimes \langle b \rangle, |a| = 8, |b| = 2, [a, b] = a^3; N_G^A = \langle a^2 \rangle \rtimes \langle b \rangle;$
- 4) $G = (H \times \langle b \rangle) \langle a \rangle, H = \langle h_1, h_2 \rangle, |h_1| = 2^k > 4, h_1^{2^{k-1}} = h_2^2, a^2 = h_1^{2^{k-2}}, h_2^{-1}h_1h_2 = h_1^{-1}, |b| = 2, [a, h_1] = a^4, [a, h_2] = b, [a, b] = a^4; N_G^A = \langle a \rangle \rtimes \langle b \rangle;$
- 5) $G = \langle y \rangle \langle b \rangle, |y| = 2^k, k \geq 4, |b| = 2^m, m \geq 2, Z(G) = \langle y^{2^m} \rangle, [y, b] = y^{2^{k-m}s}, (s, 2) = 1; N_G^A = \langle y^{2^{k-m}} \rangle \rtimes \langle b \rangle.$

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Sheffer's criterion on Appell polynomials

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The necessary and sufficient conditions of being an Appell polynomial were given in [1].

Theorem 1. *Let $\{P_n(x)\}, n=0,1,\dots$, be an arbitrary Appell sequence with the generating function $e^{tx}A(t)=\sum_{n=0}^{\infty} P_n(x)t^n$. Define the formal*

power series by $B(t):=\frac{A'(t)}{A(t)}=\sum_{n=0}^{\infty} b_n t^n$. Then every $P_n(x), n=0,1,\dots$,

satisfies the linear differential equation of infinite order

$$L[P_n(x)]=(b_0(x)+x)P'_n(x)+b_1P''_n(x)+b_2P'''_n(x)+\dots = nP_n(x). \quad (1)$$

Conversely, if $\{P_n(x)\}$ satisfies (1), then (to within multiplicative constants) $\{P_n(x)\}$ is an Appell sequence with generating function $B(t)$.

Theorem 2. *A necessary and sufficient condition for an Appell sequence $\{P_n(x)\}$, with the transfer function $A(t)$, to satisfy a finite order equation of type (1) is that $A(t) = e^{Q(t)}$, where $Q(t)$ is a polynomial; the minimum order of all equations of type (1) satisfied by $P_n(x)$ is $\text{deg}(Q(t))$.*

We specify both results in the following way.

Theorem 3. *(i) An arbitrary Appell sequence $\{P_n(x)\}$ has the exponential transfer function $A(t) = e^{Q(t)}$, where $\text{deg}(Q(t)) = k$, if and only if every $P_n(x)$ satisfies the k -th order differential equation*

$$nP_n(x)=(b_0(x)+x)P'_n(x)-b_1P''_n(x)-b_2P'''_n(x)-\dots -b_{k-1}P_n^{(k)}(x), \quad (2)$$

where not all b_i are identically equal to 0.

(ii) An arbitrary Appell sequence $\{P_n(x)\}$ has the transfer function in the form of the formal power series $A(t):=A_0+A_1t+A_2t^2+\dots$ if and only if every $P_n(x)$ satisfies the k -th order differential equation (2) with

$$b_i = \frac{1}{A_0^{i+1}} \begin{vmatrix} (i+1)A_{i+1} & A_1 & A_2 & A_3 & \dots & A_i \\ (i)A_i & A_0 & A_1 & A_2 & \dots & A_{i-1} \\ (i-1)A_{i-1} & 0 & A_0 & A_1 & \dots & A_{i-2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 2A_2 & 0 & 0 & 0 & \dots & A_1 \\ A_1 & 0 & 0 & 0 & \dots & A_0 \end{vmatrix}.$$

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On locally compact groups with zero

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We study algebraic properties on a group G such that if the discrete group G has these properties then every locally compact shift continuous topology on G with adjoined zero is either compact, or discrete. We introduce electorally flexible and electorally stable groups and establish their properties. In particular, we prove that every group with an infinite cyclic subgroup of an infinite index and every uncountable commutative group are electorally flexible, and show that every countable locally finite group is electorally stable. The main result of the paper is the following: if G is a discrete electorally flexible group then every Hausdorff locally compact shift-continuous topology on G with adjoined zero is either compact, or discrete. Also, we construct a non-discrete non-compact Hausdorff locally compact shift-continuous topology on any discrete virtually cyclic group (and hence on an electorally stable group) G with adjoined zero.

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Algorithm for LDU-factorization in domain

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Let R be a commutative domain, F its field of quotients. Let $n = 2^k$, $A \in R^{n \times n}$ be a matrix of rank r , $r \leq n$.

Definition 1. An equality $A = LDU$ is called *LDU-factorization* if matrices $L, U \in R^{n \times n}$ of rank n , (L is low triangular, U is upper triangular), matrix D of rank r has r non-zero elements equal $(det_1)^{-1}$, $(det_1 det_2)^{-1}$, ..., $(det_{r-1} det_r)^{-1}$. Here det_r is the $r \times r$ nonzero minor of A , whose position is determined by r nonzero rows and columns of the matrix D , the determinants of successively nested nondegenerate submatrices of orders $r - 1, \dots, 2, 1$ are denoted by $det_{r-1}, \dots, det_2, det_1$.

Definition 2. The matrix mapping $D \rightarrow D^{-1}$ induced by the homomorphism $F^* \rightarrow 1$ is called unit mapping.

A matrix mapping in which a submatrix of the intersection of zero rows and zero columns is replaced by a unit submatrix we denote: $D \rightarrow \hat{D}$.

Let $A_i = L_i D_i U_i$ is a *LDU-factorization*. We denote matrices: $d_i = det_r^{-1} \cdot \hat{D}_i$, $E_i = D_i^{-1}$, $I_i = E_i E_i^T$, $J_i = E_i^T E_i$, $\bar{I}_i = I - I_i$, $\bar{J}_i = I - J_i$.

LDU-factorization algorithm: $(L, D, U, M, d, W, \alpha_r) = \mathbf{LDU}(A, \alpha)$

- (1) If $A = 0$, then $D = 0$, $d = M = W = L = U = I$, $\alpha_r = \alpha$.
- (2) If $n = 1$ then $D = \frac{1}{\alpha A}$, $d = \frac{1}{\alpha A^2}$, $L = U = A, M = W = \alpha A, \alpha_r = A$.

- (3) If $n \geq 2$ we can get 4 equal blocks of matrix: $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$.

Let $(L_{11}, D_{11} U_{11}, M_{11}, d_{11}, W_{11}, \alpha_k) = \mathbf{LDU}(A_{11}, \alpha)$,

$$A''_{12} = \bar{I}_{11} M_{11} A_{12}, A''_{21} = A_{21} W_{11} \bar{J}_{11},$$

$$(L_{21}, D_{21}, U_{21}, M_{21}, d_{21}, W_{21}, \alpha_l) = \mathbf{LDU}(A''_{21}, \alpha_k), \lambda = \frac{\alpha_l}{\alpha_k} \in \mathbf{R},$$

$$(L_{12}, D_{12}, U_{12}, M_{12,12}, W_{12}, \alpha_m) = \mathbf{LDU}(A''_{12}, \alpha_k), \alpha_s = \lambda \alpha_m,$$

$$A'''_{22} = \alpha_k^2 A_{22} - A_{21} W_{11} D_{11} M_{11} A_{12}, A'''_{22} = \alpha_l^{-1} \alpha_k^{-2} \lambda \bar{I}_{21} M_{21} A'_{22} W_{12} \bar{J}_{12},$$

$$(L_{22}, D_{22} U_{22}, M_{22}, d_{22}, W_{22}, \alpha_r) = \mathbf{LDU}(A'''_{22}, \alpha_s), \text{ then}$$

$$L = \begin{bmatrix} L_{11} L_{12} & 0 \\ L_3 & L_{21} L_{22} \end{bmatrix}, D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, U = \begin{bmatrix} U_{21} U_{11} & U_2 \\ 0 & U_{22} U_{12} \end{bmatrix},$$

$$d = (\alpha_r^{-1}) \hat{D}, M = d^{-1} L^{-1}, W = U^{-1} d^{-1}, \text{ with}$$

$$U_2 = \alpha_k^{-1} J_{11} M_{11} A_{12} + \alpha_k^{-2} \alpha_l^{-1} J_{21} M_{21} A'_{22},$$

$$L_3 = \alpha_k^{-1} A_{21} W_{11} I_{11} + \alpha_k^{-2} \alpha_l^{-1} \alpha_s^{-1} \bar{J}_{21} M_{21} A'_{22} W_{12} \bar{I}_{12}, D_{12} = \lambda^{-2} D_{12},$$

$$J_{12} = \lambda J_{12} + \bar{J}_{12}, I_{12} = \lambda I_{12} + \bar{I}_{12}, L_{12} = L_{12} I_{12}, U_{12} = J_{12} U_{12},$$

$$d_{12} = \lambda^{-1} I_{12}^{-2} d_{12}, M_{12} = \lambda J_{12}^\lambda M_{12}, W_{12} = \lambda W_{12} I_{12}^\lambda.$$

Theorem 1. *The algorithm is correct and $M, W \in R^{n \times n}$.*

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Asymptotic torsion and cotorsion

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In a series of recent papers [1–3] we used stable functors on module categories to introduce new concepts of torsion, cotorsion, and asymptotic stabilization of the tensor product. The latter construct has features similar to Tate homology.

In this talk I will show how to fuse these tools together and define asymptotic torsion and cotorsion. In the case when the ring, viewed as a module over itself, has an injective resolution all of whose terms are FP_∞ (i.e., each term has a resolution by finitely generated projectives) we give a complete description of the right-derived functors of asymptotic torsion. In particular, this result applies to all artin algebras. The advantage of our approach is that the resulting functors are finitely presented, which opens a way to formal computations. The proof is based on the fact that the category of finitely presented functors is cocomplete. (Caution: colimits are not computed componentwise!)

Our main technical tool is the defect of a finitely presented functor. Applied to such a functor, it yields an easily computed module. When it is itself viewed as a functor, the defect has two adjoints and thus preserves limits and colimits. Similar to the non-stabilized case, we show that the defect of the asymptotic torsion is the asymptotic cotorsion of the ring (on the other side).

This talk does not assume familiarity with functor categories. All key concepts will be defined and explained.

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Prime semirings derivations

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Let R be a nonempty set, and let $+$ and \cdot be binary operations on R . An algebraic system $(R, +, \cdot)$ is called a *semiring* if $(R, +, 0)$ is a commutative monoid, (R, \cdot) is a semigroup and multiplication distributes over addition from either side. A semiring $(R, +, \cdot)$ is said to be *commutative* if \cdot is commutative on R .

An element $a \in R$ is called *additively cancellable* if $a + b = a + c$ follows $b = c$ for all $b, c \in R$. A semiring R is called *additively cancellative* if $K^+(R) = R$, where $K^+(R)$ is the set of all additively cancellable elements of R . Let $V(R)$ be the set of all additively invertible elements of R .

A *prime ideal* of R is an ideal $P \neq R$ such that whenever $IJ \subseteq P$ for any ideals I and J of R then either $I \subseteq P$ or $J \subseteq P$. A semiring R is called *prime* if (0) is prime. A semiring R is called *2-torsion-free* if $2r = 0$ follows $r = 0$ for any $r \in R$.

A map $\delta: R \rightarrow R$ is called a *derivation* [2] on R if $\delta(a + b) = \delta(a) + \delta(b)$ and $\delta(ab) = \delta(a)b + a\delta(b)$ for any $a, b \in R$. A semiring R equipped with a derivation δ is called *differential* with respect to the derivation δ , or a *δ -semiring*, and denoted by (R, δ) [1].

Theorem 1. *If R is a prime 2-torsion-free additively cancellative semiring, $\delta_1: R \rightarrow R$ and $\delta_2: R \rightarrow R_2$ are semiring derivations such that $\delta\delta_2$ is a derivation on R , and $\delta_1\delta_2 \in V(R)$ for all $r, s \in R$, then $\delta_1 = 0$ or $\delta_2 = 0$.*

Theorem 2. *Let R be a prime 2-torsion-free additively cancellative semiring, let $\delta: R \rightarrow R$ be a semiring derivation such that $[r, \delta(r)] = 0$ for all $r \in R$. Then R is commutative or $\delta = 0$.*

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On semiscalar equivalence of polynomial matrices over a field

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Let \mathbb{F} be an infinite field and let $\mathbb{F}_n[x]$ be the set of $n \times n$ matrices over the polynomial ring $\mathbb{F}[x]$.

A nonsingular matrix $A(x) \in \mathbb{F}_n[x]$ is semiscalar equivalent to the lower triangular matrix (see [1]) $S_l(x) = PA(x)Q(x) = [s_{ij}]$, where $P \in GL(n, \mathbb{F})$, $Q(x) \in GL(n, \mathbb{F}[x])$ and

- (a) $s_{ii}(x) = s_i(x)$, $i = 1, 2, \dots, n$, where $s_1(x)|s_2(x)|\dots|s_n(x)$ (divides) are the invariant factors of $A(x)$;
- (b) $s_{ii}(x)$ divides $s_{ji}(x)$ for all i, j with $1 \leq i < j \leq n$.

It may be noted that the matrix $S_l(x)$ is defined not uniquely with respect to the semiscalar equivalence for nonsingular matrix $A(x)$. The canonical forms with respect to semiscalar equivalence for a matrix pencil $A_0x + A_1 \in \mathbb{F}_k[x]$, where A_0 is nonsingular, were investigated in [2] and [3]. The main result of this report is the following theorem.

Theorem. *Let $A(x) \in \mathbb{F}_n[x]$ be a nonsingular matrix with the Smith normal form $S_A(x) = \text{diag}(1, a(x), \dots, a(x))$, where $a(x) \in \mathbb{F}[x]$ is a monic polynomial of degree n . The matrix $A(x)$ is semiscalar equivalent to the matrix*

$$S_l = \begin{bmatrix} 1 & 0 & \dots & \dots & \dots & 0 \\ x & a(x) & 0 & \dots & \dots & 0 \\ x^2 & 0 & a(x) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x^{n-1} & 0 & \dots & \dots & 0 & a(x) \end{bmatrix}$$

if and only if $A(x) = B(x)W(x)$, where $W(x) \in GL(n, \mathbb{F}[x])$ and $B(x) \in \mathbb{F}_n[x]$ is a regular polynomial matrix of degree $(n - 1)$.

The matrix S_l is uniquely defined for $A(x)$.

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Criterion of finite-state characteristic of transitive-stable automorphisms of a binary rooted tree

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Definition 1. Let the automorphism t have the form $t = (t_1, t_2)$ or $t = (t_1, t_2) \circ \sigma$. Let us define the functions $\pi_0(t) = t_1$ and $\pi_1(t) = t_2$.

Let us recursively denote the set W_t for automorphism $x \in \text{Aut}T_2$.

Definition 2. The identical automorphism id belongs to W_t . Let automorphism a belong to W_t . Then the automorphism $t \circ \pi_1(a)$ belongs to W_t .

Definition 3. Let us call the automorphism $t \in \text{Aut}T_2$ weakly regular if the set W_t is finite.

Theorem 1. *Let b be a transitive-stable automorphism and automorphism t is a left projection of the automorphism b . The automorphism b is finite-state if and only if t is weak regular.*

Constructive description of weakly regular automorphisms is given below.

Let us denote the operation \oplus as modulo 2 addition.

Definition 4. Denote the function $\omega : \text{Aut}T_2 \rightarrow Z_2$, $\omega(a) = \dots \alpha_2 \alpha_1 \alpha_0$, which is defined by the following relations: $\alpha_0 = 1$, $\dots \beta_2 \beta_1 \beta_0 = a(\dots \alpha_2 \alpha_1 \alpha_0)$, $\alpha_{n+1} = \alpha_n \oplus \beta_n$.

Lemma 2. *The set W_a consists of automorphisms of the form*

$$b_n = \prod_{i=0}^n a_{\omega(a)(n)}$$

Example 1. The automorphism $t' : x \rightarrow 3x$ is not regular, the automorphism id is weakly regular. Therefore, automorphism $t = (3x, id)$ is a weakly regular automorphism that is not regular.

We have an example of two transitive-stable finite-state automorphisms not conjugated in $F\text{Aut}T_2$: automorphism $\varepsilon = (id, \varepsilon) \circ \sigma$ and 11-state automorphism $a = ((3x, id), (\frac{1}{3}x, id) \circ a) \circ \sigma$ given by the ratios $a = (b, e) \circ \sigma$, $b = (c, d)$, $e = (f, e) \circ \sigma$, $d = (d, d)$, $f = (d, g)$, $c = (c, c_1)$, $c = (c, c_2) \circ \sigma$, $c_2 = (c_1, c_2)$, $g = (g_1, g_2) \circ \sigma$, $g_1 = (g, g_1)$, $g_2 = (g_2, g)$.

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Dimensions of self-similar groups

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We will talk about topological and conformal dimension of contracting self-similar groups and their connections to algebraic properties of the groups such as presentations, word problem, and amenability.

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Metric dimension of ultrametric spaces

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For an arbitrary finite metric space (X, d) a subset A , $A \subset X$, is called a resolving set (see [1]) if for any two points x and y from the space X there is an element a from subset A , such that the distances $d(a, x)$ and $d(a, y)$ are different. The metric dimension $md(X)$ of the space X is the minimum cardinality of a resolving set.

It is well known that the problem of finding the metric dimension of a metric space is NP-complete [2]. For this reason, it is interested to study classes of metric spaces with polynomial-time metric dimension.

A metric space (X, d) is called ultrametric if and only if, for arbitrary x, y, z from the set X , the following inequality holds:

$$d(x, y) \leq \max(d(x, z), d(y, z)).$$

Theorem 1. *Let (X, d) be a finite ultrametric metric space. Then there is a polynomial-time algorithm for determining the metric dimension of (X, d) .*

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Honey encryption applied to Schnorr signature scheme

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The concept of honey encryption as a general cryptographic approach to protect data was introduced in [1]. Since then a few applications and specific realizations were suggested.

We apply honey encryption to protect the secret key $a \in \mathbb{Z}_q$ in Schnorr signature scheme. Reliability of such construction could be proved with a following:

Theorem 1. *Let Schn-HE be a Honey encryption scheme applied to Schnorr signature system. For any attacker \mathcal{A}*

$$\mathbf{Adv}_{HE}(\mathcal{A}) \leq \frac{1}{q-1} \left(1 - \frac{2}{3l}\right)^{t-2},$$

where $q \in [2^{l-1}, 2^l)$.

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Commutators of some elements over divisions rings

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The residual and fixed submodules of the element σ of the complete linear group $GL(n, V)$ of the module V over the ring R are defined as the image and the kernel of the element $\sigma - 1$ and are denoted by $R(\sigma)$ and $P(\sigma)$, respectively.

Let R be an division ring. An element τ of the group $GL(n, V)$ is called transvection if $R(\tau) \subseteq P(\tau)$ and $\dim R(\tau) = 1$.

Theorem 1. *Let R be a division ring, $\text{char} R \neq 2$, $g \in GL(n, V)$, $n \geq 2$, τ be transvection such that the commutator $[g, \tau]$ commutes with g . Then $[g, \tau]$ is a unipotent element whose unipotency height does not exceed 2, which commutes with τ and $R([g, \tau]) \subseteq P(\tau) \cap P(g)$, $R(\tau) + R(g) \subseteq P([g, \tau])$.*

Theorem 2. *Let R be a division ring, $g \in GL(n, V)$, $n \geq 2$, g be a unipotent element, τ be a transvection such that $\dim(R(\tau) + R(g)) \leq 2$ and a commutator $[g, \tau]$ commutes with τ . Then $[g, \tau]$ commutes with g and inclusions take place*

$$R([g, \tau]) \subseteq P(\tau) \cap P(g), R(\tau) + R(g) \subseteq P([g, \tau]).$$

Theorem 3. *Let R be a division ring, $g \in GL(n, V)$, $n \geq 2$, τ be transvection such that $[g, \tau]$ is a unipotent element. Then the commutator $[g, \tau, \tau] = [[g, \tau], \tau]$ commutes with τ and $[g, \tau]$ and inclusions take place*

$$R([g, \tau, \tau]) \subseteq P(\tau) \cap P([g, \tau]), R(\tau) + R([g, \tau]) \subseteq P([g, \tau, \tau]).$$

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Nil restricted Lie algebras of oscillating intermediate growth

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Different versions of Burnside Problem ask what one can say about finitely generated periodic groups under additional assumptions. For associative algebras, Kurosh type problems ask similar questions about properties of finitely generated nil (more generally, algebraic) algebras. Similarly, one considers finitely generated restricted Lie algebras with a nil p -mapping.

Also, the question "how algebras grow" became popular [1]. Groups of oscillating growth were constructed in [2]. Now we study growth in the class of nil restricted Lie algebras.

Theorem 1 ([3]). *For any prime p we construct a family of 3-generated restricted Lie algebras of intermediate oscillating growth, called Phoenix algebras, they have properties:*

- *For infinitely many integers n , the algebra is "almost dying" by having a "quasi-linear" growth of type $n(\ln^{(q)} n)^k$.*
- *For infinitely many integers n the algebra is "resuscitating" by having intermediate growth of type $\exp(n/(\ln n)^\lambda)$, where λ is a constant.*
- *The growth is oscillating between these two types of functions.*
- *These restricted Lie algebras have a nil p -mapping.*

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Solvability of the matrix Sylvester-type equation in the ring of the block triangular matrices

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Let R be an adequate ring. We will denote by $M(n, R)$ the ring of $n \times n$ matrices over R and by $BT(n_1, \dots, n_k, R)$ its subring of the block upper triangular matrices $T = \text{triang}(T_{11}, \dots, T_{kk}) = [T_{ij}]_1^k$, where $T_{ij} = 0$ if $i > j$, $T_{ii} \in M(n_i, R)$, $i = 1, \dots, k$. The block matrices arise in various branches of mathematics, such as the theory of stability [1]. We described up to the association the factorizations of the matrices in the ring $BT(n_1, \dots, n_k, R)$ [2]. The description of the factorization of the block triangular matrices is reduced to the factorization of their diagonal blocks and the solving of the matrix Sylvester-type equations

$$AX + YB = C. \tag{1}$$

Roth's theorem, which establishes a connection between the solvability of the equation (1) and the equivalence of the block triangular and the block diagonal matrices, is well known [3].

If the matrix equation (1) is solvable in the ring $M(n, R)$, it may or may not be solvable in its subring $BT(n_1, \dots, n_k, R)$. We establish conditions for existence of the solutions of the matrix equation (1) in the ring $BT(n_1, \dots, n_k, R)$ and propose the way of their construction.

Theorem 1. *Let in the matrix equation (1) the matrix-coefficients A, B and $C \in BT(n_1, \dots, n_k, R)$. If matrices $\begin{bmatrix} A_{ii} & C_{ii} \\ 0 & B_{ii} \end{bmatrix}$ and $\begin{bmatrix} A_{ii} & 0 \\ 0 & B_{ii} \end{bmatrix}$ are equivalent for all $i = 1, \dots, k$ and if $(\det A_{ii}, \det B_{i+j, i+j}) = 1$ for all $i = 1, \dots, k - 1, j = 1, \dots, k - i$, then the matrix equation (1) is solvable and it has the solution $X, Y \in BT(n_1, \dots, n_k, R)$.*

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On ternary top-quasigroups whose group of parastrophic symmetries is D_8

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An algebra $(Q; f, f_1, f_2, f_3)$ is a *ternary quasigroup* and the operation f is *invertible*, if

$$\begin{aligned} f(f_1(x, y, z), y, z) &= x, & f(x, f_2(x, y, z), z) &= y, & f(x, y, f_3(x, y, z)) &= z, \\ f_1(f(x, y, z), y, z) &= x, & f_2(x, f(x, y, z), z) &= y, & f_3(x, y, f(x, y, z)) &= z. \end{aligned}$$

A triplet (f_1, f_2, f_3) of functions is called *orthogonal*, if the system

$$\begin{cases} f_1(x, y, z) = a, \\ f_2(x, y, z) = b, \\ f_3(x, y, z) = c, \end{cases}$$

has a unique solution for all a, b, c . A σ -*parastrophe* of f is defined by

$$\sigma f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} \Leftrightarrow f(x_1, x_2, x_3) = x_4, \quad \sigma \in S_4.$$

The set $\{\sigma \mid \sigma f = f\}$ is called the *group of parastrophic symmetries* of the quasigroup $(Q; f, f_1, f_2, f_3)$. A quasigroup $(Q; f, f_1, f_2, f_3)$ is called:

- a *group isotope*, if there is a group $(G; \cdot)$ and bijections $\alpha, \beta, \gamma, \delta$ such that

$$f(x, y, z) = \delta(\alpha(x) \cdot \beta(y) \cdot \gamma(z)).$$

- a *top-quasigroup*, if each triplet of its different parastrophes are orthogonal.

Theorem 1. *A ternary group isotope $(Q; f, f_1, f_2, f_3)$, whose group of parastrophic symmetries is D_8 , is a top-quasigroup if and only if there exists an abelian group $(Q, +)$, its involutive automorphism θ and an element $a \in Q$ such that $\theta + \iota$ is an automorphism of $(Q, +)$ and*

$$f(x_1, x_2, x_3) = \theta x_1 + \theta x_2 - x_3 + a.$$

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Cohomologies of Kleinian 4-group

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It is a joined work with Yuriy Drozd published in [1].

We calculate cohomologies of lattices over the Kleinian 4-group $G = \langle a, b \mid a^2 = b^2 = 1, ab = ba \rangle$. The result depends on the position of the lattice in the Auslander–Reiten quiver described in [2].

All lattices from the preprojective-preinjective component are of the form $M^k = \tau^k M$, where τ is the Auslander–Reiten translate and $M \in \{A, L_{uv} \mid u, v \in \{+, -\}\}$. Here A is the minimal overring of $\mathbb{Z}G$ and $L_{uv} = \mathbb{Z}$, where a acts as $u1$ and b acts as $v1$.

The lattices T_k^f in *homogeneous tubes* are parametrized by unital irreducible polynomial $f \notin \{x, x - 1\}$ over \mathbb{F}_2 and an integer k . There are also 3 *special tubes* with the lattices $T_k^{j^i}$, where $j \in \{2, 3, 4\}, i \in \{1, 2\}, k \in \mathbb{N}$ (see [1; 2]).

Theorem 1.

$$\hat{H}^n(G, A^k) = \begin{cases} (n - k + 1)\mathbb{F}_2 & \text{if } n \geq k, \\ (k - n)\mathbb{F}_2 & \text{if } n < k; \end{cases}$$

$$\hat{H}^n(G, L_{++}^k) = \begin{cases} (|n - k|/2 + 1)\mathbb{F}_2 & \text{if } n - k \neq 0 \text{ is even,} \\ (|n - k|/2)\mathbb{F}_2 & \text{if } n - k \text{ is odd,} \\ \mathbb{Z}/4\mathbb{Z} & \text{if } n = k \end{cases}$$

and if $(u, v) \neq (+, +)$, then

$$\hat{H}^n(G, L_{uv}^k) = (|n - k| + 1)/2\mathbb{F}_2;$$

$$\hat{H}^n(G, T_k^f) = kd\mathbb{F}_2, \text{ where } d = \deg f,$$

and for every $j \in \{2, 3, 4\}$

$$\hat{H}^n(G, T_{2k}^{j^i}) = k\mathbb{F}_2 \text{ for both } i = 1, 2,$$

$$\hat{H}^n(G, T_{2k-1}^{j^1}) = \begin{cases} k\mathbb{F}_2 & \text{if } |n| \text{ is even,} \\ (k - 1)\mathbb{F}_2 & \text{if } |n| \text{ is odd;} \end{cases}$$

$$\hat{H}^n(G, T_{2k-1}^{j^2}) = \begin{cases} k\mathbb{F}_2 & \text{if } |n| \text{ is odd,} \\ (k - 1)\mathbb{F}_2 & \text{if } |n| \text{ is even.} \end{cases}$$

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Sharpening of lower bound on elements order in finite field extensions generated by a root of unity

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Finite field extensions based on cyclotomic polynomials (generated by a root of unity) [1; 4] are constructed as follows. Let $r \geq 3$ be a prime number coprime with q (a power of prime number p) and the multiplicative order of q modulo r equals $r - 1$. Set $F_q(\theta) = F_q[x]/\Phi_r(x)$, where $\Phi_r(x) = x^{r-1} + x^{r-2} + \dots + 1$ is the cyclotomic polynomial and θ is the coset of x modulo $\Phi_r(x)$. Clearly $\theta^r = 1$. The element $\theta + \theta^{-1}$ is called a Gauss period of type $((r-1)/2, 2)$. A partition of an integer C is a sequence of non-negative integers u_1, \dots, u_C such that $\sum_{j=1}^C j u_j = C$. $U(C, d)$ is the number of such partitions of C , for which $u_1, \dots, u_C \leq d$.

The problem of finding lower bounds on the order of Gauss period and some similar elements in the extensions was in particular considered in [1; 5; 6]. It was proved [5] that $\theta + \theta^{-1}$ has the order at least $U(r - 2, p - 1)$. Theorem 1 below sharpens the previous lower bound.

Theorem 1. *Element $\theta + \theta^{-1}$ has the multiplicative order at least the number of solutions of the linear Diophantine inequality*

$$u_0 + \sum_{j=1}^{r-2} j u_j \leq r - 2$$

with conditions $0 \leq u_0$ and $0 \leq u_1, \dots, u_{r-2} \leq p - 1$.

Using known estimates from [2; 3] in the case $r \geq p^2 + 2$, we obtain

$$U(r - 2, p - 1) > \left(\frac{p(p - 1)}{160(r - 2)} \right)^{\sqrt{p}} \exp \left(\pi \sqrt{\frac{2}{3} \left(1 - \frac{1}{p} \right) (r - 2)} \right).$$

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Amalgamated free products of finite cyclic groups and finite automata

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Since countable free groups are linear they can be generated by finite initial automata over finite alphabet (see [1]). Amalgamated free products of finite groups are virtually free (see the proof of Theorem 2 in [2]). Hence amalgamated free products of finite groups can be generated by finite initial automata over finite alphabet. However, only few explicit constructions of such automata are known.

Fix some $r \geq 2$. Let l_1, \dots, l_r, k be positive integers, such that $k|l_1, \dots, k|l_r$ and $1 \leq k < l_1 \leq \dots \leq l_r$. Then for some positive integers m_1, \dots, m_r we have $km_i = l_i, 1 \leq i \leq r$.

The free product of cyclic groups of orders l_1, \dots, l_r amalgamated on the cyclic group of order k is well-defined. Denote this group by $G(l_1, \dots, l_r, k)$. The group $G(l_1, \dots, l_r, k)$ has the following presentation in terms of generators and defining relations:

$$G(l_1, \dots, l_r, k) = \langle a_1, a_2, \dots, a_r \mid a_1^{l_1} = e, \dots, a_r^{l_r} = e, a_i^{m_i} = a_j^{m_j}, i \neq j \rangle.$$

Let $L = l_1 \dots l_r$ and X be an alphabet of cardinality L . We have the following result.

Theorem 1. *The group $G(l_1, \dots, l_r, k)$ is generated by initial automata with 4 inner states over alphabet X .*

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On similarity of families of 2×2 matrices over a field

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Let \mathbb{F} be an infinite field. We refer to [1] for the definition of the complexity of similarity of matrices over a field. Two families of 2×2 matrices $\mathbf{A} = \{A_1, A_2, \dots, A_r\}$ and $\mathbf{B} = \{B_1, B_2, \dots, B_r\}$ over the field \mathbb{F} are said to be similar if there exists a matrix $T \in GL(2, \mathbb{F})$ such that $A_i = TB_iT^{-1}$ for all $i = 1, 2, \dots, r$.

The families \mathbf{A} and \mathbf{B} we associate with monic matrix polynomials $A(\lambda) = I_2\lambda^r + \sum_{i=1}^r A_i\lambda^{r-i}$ and $B(\lambda) = I_2\lambda^r + \sum_{i=1}^r B_i\lambda^{r-i}$ over \mathbb{F} of degree r respectively, where I_2 is the identity 2×2 matrix. Thus, the families \mathbf{A} and \mathbf{B} are similar over the field \mathbb{F} if and only if the monic matrix polynomials $A(\lambda)$ and $B(\lambda)$ are similar over \mathbb{F} . The purpose of this report is to give a criterion of similarity of two families of 2×2 matrices over the field \mathbb{F} .

It is clear, if matrices $A(\lambda)$ and $B(\lambda)$ are similar over \mathbb{F} , then their Smith normal forms are the same. Let the monic matrix polynomials $A(\lambda)$ and $B(\lambda)$ be equivalent and $S(\lambda) = \text{diag}(s_1(\lambda), s_2(\lambda))$ be their Smith normal form. For $A(\lambda)$ and $B(\lambda)$ there exist matrices $P_1, P_2 \in GL(2, \mathbb{F})$ and $Q_1(\lambda), Q_2(\lambda) \in GL(2, \mathbb{F}[\lambda])$ such that

$$P_1A(\lambda)Q_1(\lambda) = s_1(\lambda) \begin{bmatrix} 1 & 0 \\ a(\lambda) & s(\lambda) \end{bmatrix}$$

and

$$P_2B(\lambda)Q_2(\lambda) = s_1(\lambda) \begin{bmatrix} 1 & 0 \\ b(\lambda) & s(\lambda) \end{bmatrix},$$

where $s(\lambda) = \frac{s_2(\lambda)}{s_1(\lambda)}$ and $\{\deg a(\lambda), \deg b(\lambda)\} < \deg s(\lambda)$ (see [2]). Put $a(\lambda)b(\lambda) = s(\lambda)q(\lambda) + c(\lambda)$, where $\deg c(\lambda) < \deg s(\lambda)$. On the basis of the above considerations we obtain the following theorem.

Theorem. *The families of 2×2 matrices \mathbf{A} and \mathbf{B} are similar over \mathbb{F} if and only if there exist linearly independent vectors $X_1 = [x_1 \ x_2]$ and $X_2 = [x_3 \ x_4]$ over \mathbb{F} such that $X_1 \begin{bmatrix} a(\lambda) \\ c(\lambda) \end{bmatrix} + X_2 \begin{bmatrix} 1 \\ b(\lambda) \end{bmatrix} = 0$.*

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The prescribed Ricci curvature problem on homogeneous spaces

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We will discuss the problem of recovering the “shape” of a Riemannian manifold M from its Ricci curvature. After reviewing the relevant background and the history of the subject, we will focus on the case where M has a large symmetry group. Based on joint work with Mark Gould (The University of Queensland) and Wolfgang Ziller (The University of Pennsylvania).

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***D*-center and *D*-derived subalgebra of Leibniz algebras**

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Let L be a left Leibniz algebra over a field F and D be a subalgebra of algebra of derivations $\text{Der}(L)$. Put $\text{Ann}_L(D) = \bigcap_{\alpha \in D} \text{Ker}(\alpha)$ and $[L, D] = \sum_{\alpha \in D} \text{Im}(\alpha)$.

Let $a \in L$. Define the mapping $l_a : L \rightarrow L$ by $l_a(x) = [a, x]$. Note that l_a is a derivation of L and the set $\text{Ad}^l(L) = \{l_a \mid a \in L\}$ is an ideal of $\text{Der}(L)$. If $\text{Ad}^l(L) \leq D$, then $\text{Ann}_L(D) \leq \text{Ann}_L(\text{Ad}^l(L)) = \zeta^r(L)$. Thus

$$\text{Ann}_L(D) \cap \zeta^l(L) \leq \text{Ann}_L(\text{Ad}^l(L)) \cap \zeta^l(L) = \zeta^r(L) \cap \zeta^l(L) = \zeta(L).$$

In particular, $\text{Ann}_L(D) \cap \zeta^l(L)$ is an ideal of L .

We will say that $A_L(D) = \text{Ann}_L(D) \cap \zeta^l(L)$ is the *D*-center of L . Note that $A_L(D) \leq \zeta(L)$ and if $D = \text{Ad}^l(L)$ then $A_L(D) = \zeta(L)$. We will say that $[L, D]$ is the *D*-derived subalgebra of L . If $D = \text{Ad}^l(L)$ then $[L, D] = [L, L]$. The main result is the following

Theorem. *Let L be a Leibniz algebra over a field F , D be a subalgebra of algebra of derivations $\text{Der}(L)$ such that $\text{Ad}^l(L) \leq D$. Suppose that $\dim_F(D/\text{Ad}^l(L)) = k$ is finite. If $\dim_F(L/A_L(D)) = t$ is finite, then $\dim_F([L, D]) \leq t(k + t)$.*

Corollary 1 [1]. *Let L be a Leibniz algebra over a field F . If $L/\zeta(L)$ has finite dimension t , then $\dim_F([L, L]) \leq t^2$.*

Corollary 2 [3]. *Let L be a Lie algebra over a field F . If $L/\zeta(L)$ has finite dimension t , then $\dim_F([L, L]) \leq t(t + 1)/2$.*

Corollary 3. *Let L be a Leibniz algebra over a field F . Suppose that $L/A_L(\text{Der}(L))$ has finite dimension t . Then $\dim_F([L, \text{Der}(L)]) \leq t(t + 1)$.*

Corollary 4 [2]. *Let L be a Lie algebra over a field F . Suppose that $L/A_L(\text{Der}(L))$ has finite dimension t . Then $\dim_F([L, \text{Der}(L)]) \leq t(t + 1)/2$.*

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Properties of the invariant factors of the sum of matrices

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The invariant factors are played an important role in the study of matrices problem [1; 3]. There are many papers that rather deeply describe the structure and properties of invariant factors in the matrix $C = AB$. However, there is little information about the invariant factors of the matrix $C = A + B$. Therefore, the study of the properties of invariant factors the matrices and their sums are relevant. Describes the divisibility of the invariant factors of the nonsingular matrices A, B and $C = A + B$ over of the principal ideal domains can be found in [4]. In this paper, those results are extended over wider classes of rings.

Let R be a commutative elementary divisor domain with $1 \neq 0$ [2], $M_n(R)$ be a ring $n \times n$ matrices over R . Consider a nonsingular matrix $A \in M_n(R)$. Since R is an elementary divisor domain there

$$A \sim E = \text{diag}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n), \quad \varepsilon_i | \varepsilon_{i+1}, \quad i = 1, \dots, n - 1.$$

The matrix E is called the Smith normal form or canonical diagonal form of matrix A , ε_i are invariant factors of matrix A .

The notation $a|b$ means that the element a divides the element b .

Theorem 1. *Let R be a commutative elementary divisor domain with $1 \neq 0$. Let $A \sim E = \text{diag}(\varepsilon_1, \dots, \varepsilon_n)$, $\varepsilon_i | \varepsilon_{i+1}$, $i = 1, \dots, n - 1$, $B \sim \Delta = \text{diag}(\delta_1, \dots, \delta_n)$, $\delta_i | \delta_{i+1}$, $i = 1, \dots, n - 1$, $C = A + B \sim \Gamma = \text{diag}(\gamma_1, \dots, \gamma_n)$, $\gamma_i | \gamma_{i+1}$, $i = 1, \dots, n - 1$. And let*

$$F = \text{diag}(\varepsilon_1, \dots, \varepsilon_n, \delta_1, \dots, \delta_n) \sim \Omega = \text{diag}(\omega_1, \dots, \omega_{2n}),$$

$$\omega_i | \omega_{i+1}, \quad i = 1, \dots, 2n - 1. \quad \text{Then } \omega_i | \gamma_i, \quad i = 1, \dots, n.$$

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Dmytro Savchuk

**The lamplighter group of rank two generated by a
bireversible automaton**

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The examples of bireversible automata that generate non-virtually free groups were first constructed quite recently in works of Bondarenko, D'Angeli, Rodaro, and Skipper, Steinberg. In this talk I will describe an explicit construction of a 4-state 2-letter bireversible automaton generating the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ of rank two. The action of the generators on the boundary of the tree can be induced by the affine transformations on the ring $\mathbb{Z}_2[[t]]$ of formal power series over \mathbb{Z}_2 . This is a joint work with Elsayed Ahmed.

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On Leibniz algebras with a dense family of ideals

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Let L be a Leibniz algebra over a field F and \mathfrak{S} be some family of subalgebras of L . We say that a family \mathfrak{S} is *dense* in L , if for every pair of subalgebras A, B of L such that $A \leq B$ and A is not maximal in B there exists a subspace $S \in \mathfrak{S}$ such that $A \leq S \leq B$ (of course, S can be coincides with A or B). The origins of this concept are in group theory. There is a whole array of articles devoted to the study of groups, having different natural families of subgroups. This topic is not limited to the framework of the classical theory of groups. For example, in [1] the infinite dimensional linear groups having a dense family of subgroups of finite central dimension were considered. A Leibniz algebra which is not a Lie algebra has one specific ideal. Denote by $\text{Leib}(L)$ the subspace generated by the elements $[a, a]$, $a \in L$. It is possible to prove that $\text{Leib}(L)$ is an ideal of L . Moreover, $L/\text{Leib}(L)$ is a Lie algebra. Conversely, if H is an ideal of L such that L/H is a Lie algebra, then $\text{Leib}(L) \leq H$. The ideal $\text{Leib}(L)$ is called the *Leibniz kernel* of L .

A Leibniz algebra L is *abelian*, if $[x, y] = 0$ for each $x, y \in L$.

Theorem 1. *Let L be a Leibniz algebra, having a dense family of ideals. If L includes an abelian subalgebra A , having dimension at least 4, then every subalgebra of L is an ideal of L .*

Theorem 2. *Let L be a Leibniz algebra, having a dense family of ideals. Suppose that $L/\text{Leib}(L)$ includes an abelian subalgebra of dimension 4. Then either every subalgebra of L is an ideal or L satisfies the following conditions: (a) $\dim_F(\text{Leib}(L)) \leq 3$; (b) L is nilpotent; (c) $L/\text{Leib}(L)$ is abelian; (d) L includes an ideal E such that $\text{Leib}(L) \leq \zeta(E)$ and $[v, v] \neq 0$ for every $v \in E$ such that $v \notin \text{Leib}(L)$; (e) if $\dim_F(\text{Leib}(L)) = 3$, then $\dim_F(L/E) \leq 3$; (f) if $\dim_F(\text{Leib}(L)) = 2$, then $\dim_F(L/E) \leq 6$; (g) if $\dim_F(\text{Leib}(L)) = 1$, then $\dim_F(L/E) \leq 8$.*

In particular, if $\dim_F(\text{Leib}(L)) = 1$, then every subalgebra of E is an ideal of L .

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Canonical form of a reduced matrix with one zero sub-diagonal element

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Let a matrix $A(x) = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ a_1(x) & x^{k_1} & 0 \\ a_3(x) & a_2(x) & x^{k_2} \end{array} \right\|$ have the following prop-

erties: (i) $\deg a_1 < k_1$, $\deg a_2$, $\deg a_3 < k_2$, $a_2(x) = x^{k_1} a_2'(x)$, $a_1(0) = a_2'(0) = a_3(0) = 0$;

(ii) $\text{codeg } a_3 \neq \text{codeg } a_1, \text{codeg } a_2'$, if $\text{codeg } a_3 < \text{codeg } a_2$;

(iii) $\text{codeg } a_3 \neq 2\text{codeg } a_1 + \text{codeg } a_2'$ and in $a_1(x)$ the monomial of the degree $2\text{codeg } a_1$ is absent, if $\text{codeg } a_3 \geq \text{codeg } a_2$.

The purpose of this report is to construct the canonical form of the matrix $A(x)$ in the class $\{PA(x)Q(x)\}$, where $P \in GL(3, \mathbb{C})$, $Q(x) \in GL(3, \mathbb{C}[x])$ (notation: $A(x) \approx PA(x)Q(x)$). If both elements $a_1(x), a_2(x)$ of the matrix $A(x)$ are non-zero, then we may take their junior coefficients to be identity elements. In the opposite case, we may take the junior coefficients of the non-zero subdiagonal elements of the matrix $A(x)$ to be one. Such matrix $A(x)$ in [1] is called *the reduced matrix*. In this report we consider the case, when some one of the elements $a_1(x), a_2(x), a_3(x)$ of the matrix $A(x)$ are equal to zero.

Theorem. *Let the elements of the reduced matrix $A(x)$ satisfy conditions: $a_3(x) \equiv 0$, $a_1(x), a_2(x) \neq 0$, $a_2(x) = x^{k_1} a_2'(x)$, and $q_1 := \text{codeg } a_1$, $q_2 := \text{codeg } a_2$. Then $A(x) \approx B(x)$, where the elements of the reduced matrix*

$$B(x) = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ b_1(x) & x^{k_1} & 0 \\ b_3(x) & b_2(x) & x^{k_2} \end{array} \right\|$$

satisfy conditions: $b_3(x) \equiv 0$, $b_1(x), b_2(x) \neq 0$, $b_2(x) = x^{k_1} b_2'(x)$ and in $b_2(x)$ there are no $(2q_2 + k_1)$ - and $(2q_2 + q_1 + k_1)$ - monomials. The matrix $B(x)$ is uniquely determined.

We have also considered other cases where $a_1(x) \equiv 0$, $a_2(x), a_3(x) \neq 0$, or $a_2(x) \equiv 0$, $a_1(x), a_3(x) \neq 0$,

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Quasigroup varieties with inverse properties

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Left, right and middle translations L_a, R_a, M_a of a quasigroup $(Q; \cdot)$ are defined by $L_a(x) := a \cdot x, R_a(x) := x \cdot a, M_a(x) := x \cdot^r a$. Therefore, there are six sets of bijections in each quasigroup:

$$\begin{aligned} \{L_x \mid x \in Q\}, & \quad \{R_x \mid x \in Q\}, & \quad \{M_x \mid x \in Q\}, \\ \{L_x^{-1} \mid x \in Q\}, & \quad \{R_x^{-1} \mid x \in Q\}, & \quad \{M_x^{-1} \mid x \in Q\}. \end{aligned} \quad (1)$$

A quasigroup is known to have an inverse property (a singly inverse property) if (exactly) two different sets from (1) coincide. A class of quasigroups is considered to have a singly inverse property if all its quasigroups have an inverse property, in addition one of them has a singly inverse property. The well-known quasigroup classes of *IP, LIP, RIP, MIP, CIP* quasigroups are among them. A variety of Moufang loops is a subvariety of the class of *IP* quasigroups. The respective classes of quasigroups are studied in [1], [2], [3].

Theorem. *There are nine quasigroup varieties with a singly inverse property and they are described by one of the following identities:*

$$\begin{aligned} yx &= z \cdot (xy \cdot^\ell z), & (z \cdot^\ell xz) \cdot xy &= y, & yx \cdot (zx \cdot^r z) &= y, \\ xy \cdot (xz \cdot^r z) &= y, & yx \cdot y &= zx \cdot z, & y \cdot xy &= z \cdot xz, \\ (zx \cdot^\ell z) \cdot y &= yx, & y \cdot yx &= z \cdot zx, & xy \cdot y &= xz \cdot z. \end{aligned}$$

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Secret sharing schemes

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Secret sharing is an alternative for secure data outsourcing. It gives an ability to distribute and share secret among the group of scheme users. Secret is possible to reconstruct only if threshold number of users are involved restoration process, while every part of a secret doesn't have any ability to reconstruct secret itself [1].

Research and analysis of working principles and main components of the basic schemes is shortly reviewed. A new approach of secret sharing scheme realization, using the Adi Shamir scheme and RSA encryption is presented. The Adi Shamir scheme has become popular since it's a basic secret sharing scheme, based on polynomial interpolation [2]. In the talk proposed new algorithm called Shamir scheme with complication, due to which the secret sharing scheme information security raises, because even if a malefactor collects threshold number of parts of a secret, the reconstruction process will be impossible without parts decryption.

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Metabelian Lie algebras of derivations of rank two

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Let \mathbb{K} be an algebraically closed field of characteristic zero and $R = \mathbb{K}(x, y)$ the field of rational functions. Recall that a \mathbb{K} -linear mapping $D : R \rightarrow R$ is called a \mathbb{K} -derivation if $D(fg) = D(f)g + fD(g)$ for all $f, g \in R$. We denote by $\widetilde{W}_2(\mathbb{K})$ the Lie algebra of all \mathbb{K} -derivations on R , this algebra is considered over the field \mathbb{K} , it is a two-dimensional vector space over R (but not a Lie algebra over R). The structure of finite dimensional subalgebras of the Lie algebra $\widetilde{W}_2(\mathbb{K})$ was studied by many authors, see for example, [1].

Using some results of the paper [2] we give a characterization of maximal (by inclusion) metabelian infinite dimensional subalgebras of $\widetilde{W}_2(\mathbb{K})$ of rank 2 over the field R .

Theorem. *Let L be a metabelian infinite dimensional over \mathbb{K} subalgebra of rank 2 over \mathbb{K} of the Lie algebra $\widetilde{W}_2(\mathbb{K})$. If L is maximal (by inclusion) with such properties, then $L = I \rtimes \mathbb{K}D_2$, where I is an abelian ideal of L of the form $I = \mathbb{K}(\varphi)D_1$ with a rational function $\varphi \in \text{Ker}D_1$, $[D_2, D_1] = \psi D_1$ for some $\psi \in \text{Ker}D_1$.*

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On generalized ternary quasigroup functional equations of the type (5; 3; 0; 0)

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Let Q be a set. A mapping $g : Q^3 \rightarrow Q$ is called a *ternary invertible function*, if there exist functions g_1, g_2, g_3 such that the identities

$$\begin{aligned} g(g_1(x, y, z), y, z) &= x, \quad g(x, g_2(x, y, z), z) = y, \quad g(x, y, g_3(x, y, z)) = z, \\ g_1(g(x, y, z), y, z) &= x, \quad g_2(x, g(x, y, z), z) = y, \quad g_3(x, y, g(x, y, z)) = z. \end{aligned}$$

hold. A *generalized ternary quasigroup functional equation of the type (5; 3; 0; 0)* means that all functional variables are pairwise different and take on their values in the set of all ternary invertible functions defined on a set Q and it has two individual variables with 5 and 3 appearances. Two equations are said to be *parastrophically-primarily equivalent* [1], if one can be got from the other by applying the equalities given above.

Theorem 1. *These equations are parastrophically-primarily equivalent to*

$$F_1(F_2(y, y, y), x, x) = F_3(x, x, x), \tag{1}$$

$$F_1(F_2(x, x, x), x, y) = F_3(x, y, y), \tag{2}$$

$$F_1(F_2(x, x, x), y, y) = F_3(x, x, y), \tag{3}$$

$$F_1(F_2(x, y, y), x, x) = F_3(x, x, y), \tag{4}$$

$$F_1(F_2(x, x, y), x, y) = F_3(x, x, y). \tag{5}$$

Let $(Q; +, 0)$ be a commutative group, $\alpha_i, \beta_i, \gamma_i$ are its automorphisms, $a_i \in Q$ and $f_i(x, y, z) := \alpha_i x + \beta_i y + \gamma_i z + a_i, i = 1, 2, 3$.

Theorem 2. *The triplet (f_1, f_2, f_3) is a solution of (1)–(5) iff*

- (1) : $a_3 = a_1 + \alpha_1 a_2, \quad \gamma_2 = -\alpha_2 - \beta_2, \quad \gamma_3 = \beta_1 + \gamma_1 - \alpha_3 - \beta_3;$
- (2) : $a_3 = \alpha_1 a_2 + a_1, \quad \beta_3 = \gamma_1 - \gamma_3, \quad \alpha_3 = \alpha_1(\alpha_2 + \beta_2 + \gamma_2) + \beta_1;$
- (3) : $a_3 = \alpha_1 a_2 + a_1, \quad \gamma_3 = \beta_1 + \gamma_1, \quad \beta_3 = \alpha_1(\alpha_2 + \beta_2 + \gamma_2) - \gamma_3;$
- (4) : $a_3 = \alpha_1 a_2 + a_1, \quad \gamma_3 = \alpha_1 \beta_2 + \alpha_1 \gamma_2, \quad \beta_3 = \alpha_1 \alpha_2 + \beta_1 + \gamma_1 - \alpha_3;$
- (5) : $a_3 = \alpha_1 a_2 + a_1, \quad \gamma_3 = \alpha_1 \gamma_2 + \gamma_1, \quad \beta_3 = \alpha_1 \alpha_2 + \alpha_1 \beta_2 + \beta_1 - \alpha_3.$

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Irreducibility of some 8×8 -matrices over local ring of finite length

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We consider a monomial $n \times n$ -matrices of the form

$$M(t, k, n) = \begin{pmatrix} 0 & \dots & 0 & 1 \\ 1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 \end{pmatrix} \cdot \text{diag}[\overbrace{1, \dots, 1}^k, t, \dots, t],$$

where k is an integer smaller than n over the commutative local ring with Jacobson's radical which is the principle ideal generated by the element t . It is known [1] that for k which is not coprime to n all $n \times n$ -matrices are reducible. If the radical is nonzero see [2; 3], then all considered $n \times n$ -matrices with $k = 1$ or $k = n - 1$ are irreducible and all considered matrices of size less than 7 are irreducible if and only if k is coprime with n . If the degree of nilpotency of radical is 2, then the matrices $M(t, 3, 7)$, $M(t, 4, 7)$ are reducible [1; 3].

If the radical is not nilpotent, or the degree of nilpotency of the radical is higher than 2, then all considered $n \times n$ -matrices for $n < 8$ are irreducible if and only if k is coprime with n [4]. It is known that $M(t, 1, 8)$, $M(t, 7, 8)$ are irreducible and $M(t, 2, 8)$, $M(t, 4, 8)$, $M(t, 6, 8)$ are reducible. We shown the irreducibility of the matrix $M(t, 3, 8)$ and $M(t, 5, 8)$.

These studies were carried out together with V. M. Bondarenko.

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Weighted Fourier algebras and complexification

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Fourier algebra $A(G)$ of a locally compact group G , introduced by Eymard, is one of the favourite objects in abstract harmonic analysis. It has an advantage to be commutative that allows one to examine its Gelfand spectrum, which is known to be topologically isomorphic to G ; the fact makes a non-trivial connection between Banach algebras and groups. We will discuss a weighted variant of Fourier algebra and show its connection with complexification of the underlying group. For compact groups this was done thanks to abstract complexification due to McKennon [Crelle, 79'] and Cartwright/McMullen [Crelle, 82']. We extended this theory to general locally compact groups and use the model to describe the Gelfand spectrum of weighted Fourier algebras, showing that the latter is a part of the complexification for a wide class of locally compact groups and weights. I shall also present different examples of weights and determine the spectrum of the corresponding algebras.

This talk is based on joint work with Olof Giselsson, Mahya Ghandehari, Hun Hee Lee, Jean Ludwig and Nico Spronk.

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On primitive modules over group algebras of linear groups

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Let G be a group, k be a field and M be a kG -module. The module M is said to be primitive if it may not be presented in the form $M = N \otimes_{kH} kG$, where $H \neq G$ is a subgroup of G and N is a kH -submodule of M . A representation of the group G over the field k is said to be primitive if the module of the representation is primitive.

The set $\Delta(G)$ of all elements $g \in G$ such that $|G : C_G(g)| < \infty$ is a characteristic subgroup of G which is said to be the FC -center of G .

In [1] Harper showed that any finitely generated not abelian-by-finite nilpotent group has an irreducible primitive representation over any not locally finite field. In [2] we proved that in the class of soluble groups of finite rank with the maximal condition for normal subgroups only polycyclic groups may have irreducible primitive faithful representations over a field of characteristic zero. It is well-known that any polycyclic group is linear and has finite rank.

Theorem 1. *Let G be a linear group of finite rank, let k be a field of characteristic zero and let M be an irreducible kG -module such that $C_G(M) = 1$. If the module M is primitive then:*

(i) $\Delta(G)$ has an infinite characteristic torsion-free subgroup C of finite index;

(ii) the annihilator $P = \text{Ann}_{kC}(M)$ is a prime G -invariant ideal of kC ;

(iii) the group G has a nilpotent normal torsion-free subgroup H such that the quotient group G/H is nilpotent-by-finite and the module M is kH/PkH -torsion-free.

In the case of a polycyclic group G , the above result follows from theorem A of [3].

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On families of stable subsemigroups and subgroups of affine Cremona semigroups with exponential growth of periods and noncommutative cryptography

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The famous Singer cycle of general affine linear group $AGL_n(F_q)$ of dimension n defined over the finite field F_q is its element of order $q^n - 1$. We search for other families of noncommutative subgroups G_n (or subsemigroups S_n) of degree bounded by constant d (stable semigroups) in the semigroup $E_n(F_q)$ of all endomorphisms $F_q[x_1, x_2, \dots, x_n]$ given via their values on $x_i, i = 1, 2, \dots, n$ such that maximal order (period) of representative of G_n (S_n) is exponential expression in variable n . Noncommutative subsemigroups of bounded degree and unbounded period can be considered in the more general case of $K[x_1, x_2, \dots, x_n]$ where K is commutative ring with unity. Natural action Δ of semigroup $\text{End}(K[x_1, x_2, \dots, x_n]) = E_n(K)$ of all endomorphisms of $K[x_1, x_2, \dots, x_n]$ on K_n defines finite affine Cremona Semigroup $CS(K^n) = \Delta(E_n(K))$.

For a family of stable subgroups $G_n(K) < E_n(K)$ of degree d we define $\text{ord}(d, G_n(K))$ as the maximal finite order of $\Delta(g), g \in G_n(K)$ and $\text{deg}(g) = d$. For a family of stable semigroups $S_n(K) \in E_n(K)$ of degree d we define $\text{per}(d, S_n(K))$ as the maximal finite period of nonbijective $\Delta(g), g \in G_n(K)$ and $\text{deg}(g) = d$.

Theorem 1. *For each commutative ring K with unity and $d \geq 2$ there is a family of noncommutative stable subgroups $G_n(K) < E_n(K)$ of degree d and stable semigroups $S_n(K) < E_n(K)$ of degree d such that $G_n(K) < S_n(K)$, $\text{ord}(d, G_n(K)) \geq n - 1$, $\text{per}(d, S_n(K)) \geq n - 2$ and in the case of $K = F_q$ inequalities $\text{ord}(d, G_n(F_q)) \geq q^{n-1} - 1$, $\text{per}(d, S_n(K)) \geq q^{n-2}$ hold.*

We consider some application of these bounds ([1],[2]).

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On noncommutative cryptography responds to challenges of quantum computer and secure big data processing

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1. Post-Quantum Cryptography (PQC) is an answer to a threat coming from a full-scale quantum computer able to execute Shor's algorithm. With this algorithm implemented on a quantum computer, currently used public key schemes, such as RSA, elliptic curve cryptosystems, other algorithms based on discrete logarithm problem are no longer secure. The U.S. NIST made a step toward mitigating the risk of quantum attacks by announcing the PQC standardisation process for already known public key algorithms which are possible resistant to attacks with the usage of quantum computers and modern deterministic computational machinery. In March 2019 NIST published a list of candidates qualified to the second round of the PQC process. Further perspectives to develop a new of post quantum secure algorithm are connected with Non-commutative cryptography, where the cryptographic primitives and systems are based on algebraic structures (platforms) like groups, semigroups and non-commutative rings.

2. We are working with platforms which are subgroups of affine Cremona groups of automorphisms of $K[x_1, x_2, \dots, x_n]$ defined over finite commutative ring K . We present new protocols which use the concept of tame homomorphism for semigroups and the complexity of word problem for Cremona semigroup about the decomposition of a given polynomial transformation g into given generators (see [1]). We use platforms which are subgroups of cubical transformations defined via families of known extremal graphs $D(n, K)$ and $A(n, K)$. Multivariate nature of collision maps allows to use these algorithms for the safe exchange of multivariate transformations. New multivariate protocols allows safe exchange of fast multivariate stream ciphers and key dependent hash functions (MACs) developed for processing of Big Data objects [2].

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Topological full group containing the Grigorchuk group

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Topological full groups provide a connection between topological dynamics and group theory. A countable transformation group is associated to a minimal dynamical system on a Cantor set and, as an abstract group, it (somehow) completely encodes the dynamics.

We consider the topological full group $[[T]]$ of a substitution subshift $T : \Omega \rightarrow \Omega$ generated by a substitution $a \rightarrow aca, b \rightarrow d, c \rightarrow b, d \rightarrow c$. This substitution (denote it σ) was originally used by Lysënok to obtain a nice recursive representation of the Grigorchuk group:

$$\begin{aligned} G &= \langle a, b, c, d \mid 1 = a^2 = b^2 = c^2 = d^2 = bcd \\ &= \sigma^k((ad)^4) = \sigma^k((adacac)^4), k \geq 0 \rangle. \end{aligned}$$

As shown by Matte Bon, the Grigorchuk group G embeds into $[[T]]$.

We study group-theoretical properties of the topological full group $[[T]]$. In particular, we show that it is finitely generated.

Elements of the phase space Ω of the subshift T are bi-infinite sequences $\dots\omega_{-2}\omega_{-1}\omega_0.\omega_1\omega_2\dots$, where each $\omega_i \in \{a, b, c, d\}$. Given finite strings u and w , let $[u.w]$ denote the set of all sequences in Ω that match the pattern $u.w$. If u and w are not both empty strings, the set $U = [u.w]$ is disjoint from $T(U)$. Hence we can define an involution $\delta_U : \Omega \rightarrow \Omega$ by $\delta_U(x) = T(x)$ if $x \in U$, $\delta_U(x) = T^{-1}(x)$ if $x \in T(U)$, and $\delta_U(x) = x$ otherwise. It is known that the group $[[T]]$ is generated by T and all transformations of the form δ_U .

Theorem 1. *The topological full group of the substitution subshift T is generated by transformations T , $\delta_{[.b]}$, $\delta_{[.d]}$, and $\delta_{[.acacac]}$.*

Theorem 2. *The subgroup of $[[T]]$ generated by $\delta_{[.a]}$, $\delta_{[.b]}\delta_{[.c]}$, $\delta_{[.c]}\delta_{[.d]}$ and $\delta_{[.d]}\delta_{[.b]}$ is isomorphic to the Grigorchuk group. The subgroup of $[[T]]$ generated by $\delta_{[.a]}$, $\delta_{[.b]}$, $\delta_{[.c]}$ and $\delta_{[.d]}$ is isomorphic to the Grigorchuk overgroup.*

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Clear elements and clear rings

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Throughout the paper we suppose R is an associative ring with non-zero unit and $U(R)$ its group of units. The vector space of matrices over the ring R of size $k \times l$ is denoted by $R^{k \times l}$ and group of units of the ring $R^{n \times n}$ by $GL_n(R)$. A ring R is called an *elementary divisor ring* [1] if for an arbitrary matrix A of order $n \times m$ over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that PAQ is a diagonal matrix (d_{ij}) for which d_{ii} is a total divisor $d_{i+1, i+1}$ (i.e. $Rd_{i+1, i+1}R \subseteq d_{ii}R \cap Rd_{ii}$) for each i . An element a of ring R is *clear* if $a = r + u$ where r is unit-regular element and $u \in U(R)$. The ring R is *clear* if every its element is such.

Theorem 1. *Let R be a commutative elementary divisor ring and A is a full nonsingular matrix of $R^{2 \times 2}$. Then exist invertible matrices $P, Q \in GL_2(R)$ such that PAQ is nontrivial clear element of $R^{2 \times 2}$.*

Theorem 2. *Let R be a commutative elementary divisor ring. Then every full nonsingular matrix $A \in R^{2 \times 2}$ is nontrivial clear.*

Theorem 3. *Let R be a semi-simple commutative Bézout domain. The next statements are equivalent:*

1. R is an elementary divisor ring;
2. any full nonsingular matrix of $R^{2 \times 2}$ is nontrivial clear.

Theorem 4. *Let R be a commutative Bezout domain. Ring $R^{2 \times 2}$ is a clear if and only if $R^{2 \times 2}$ is 2-good ring.*

Some open questions.

1. Is the commutative clear ring a ring of unit-regular stable range 1?
2. Is the notion of a ring of unit-regular stable range 1 a left-right symmetric?

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Simple element of a Bezout domain

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Let R be an associative ring with nonzero units. All the necessary definitions and facts can be found in [1].

Definition. An element $a \in R$ is called simple if $RaR = R$. If for $a \in R \setminus \{0\}$ there exist $u_1, v_1, u_2, v_2 \in R$ such that $u_1av_1 + u_2av_2 = 1$ we say that a is 2-simple.

Theorem 1. *Let R be an elementary divisor domain. Then any simple element of R is 2-simple.*

Theorem 2. *Let R be a Bezout domain with Dubrovin condition of stable range 1 in which any simple element is a finite element. Then R is an elementary divisor ring.*

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Finitely presented algebras

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We will address two questions.

- 1) Which algebras are finitely presented?
- 2) Once we know that an algebra is finitely presented, what is its time complexity?

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The least dimonoid congruence on the free g -dimonoid

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A dimonoid [1] is a nonempty set equipped with two binary associative operations \dashv and \vdash satisfying the axioms $(x \dashv y) \dashv z = x \dashv (y \vdash z)$ (D1), $(x \vdash y) \dashv z = x \vdash (y \dashv z)$ (D2) and $(x \dashv y) \vdash z = x \vdash (y \vdash z)$ (D3). A nonempty set equipped with two binary associative operations \dashv and \vdash satisfying the axioms (D1) and (D3) is called a generalized dimonoid [2] or a g -dimonoid for short.

Let X be an arbitrary nonempty set, let $F[X]$ be the free semigroup on X , and let w be an arbitrary word over X . The length of w is denoted by $l(w)$. For every $x \in X$, the number of occurrences of the element x in w is denoted by $d_x(w)$. Let further T be the free monoid on the two-element set $\{a, b\}$, let $\theta \in T$ be the empty word and $*$ denotes the operation on T . By definition, $l(\theta) = 0$. Define operations \dashv and \vdash on T by the rules $u_1 \dashv u_2 = u_1 * a^{l(u_2)+1}$, $u_1 \vdash u_2 = u_2 * b^{l(u_1)+1}$ for all $u_1, u_2 \in T$. The obtained algebra is denoted by $T_a^b(1)$ [3]. Let further $XT_a^b(1) = \{(w, u) \in F[X] \times T_a^b(1) \mid l(w) - l(u) = 1\}$. By Theorem 1 of [3], $XT_a^b(1)$ is the free g -dimonoid.

If ρ is a congruence on a g -dimonoid (D, \dashv, \vdash) such that $(D, \dashv, \vdash)/\rho$ is a dimonoid, we say that ρ is a dimonoid congruence. If ρ is a congruence on a g -dimonoid (D, \dashv, \vdash) such that operations of $(D, \dashv, \vdash)/\rho$ coincide, we say that ρ is a semigroup congruence.

Theorem 1. *Let $XT_a^b(1)$ be the free g -dimonoid, $(w_1, u_1), (w_2, u_2) \in XT_a^b(1)$.*

(i) *Define a relation \tilde{h} on $XT_a^b(1)$ by*

$$(w_1, u_1)\tilde{h}(w_2, u_2) \quad \text{if and only if} \quad w_1 = w_2 \quad \text{and} \quad d_b(u_1) = d_b(u_2).$$

Then \tilde{h} is the least dimonoid congruence on $XT_a^b(1)$.

(ii) *Define a relation $\tilde{\gamma}$ on $XT_a^b(1)$ by $(w_1, u_1)\tilde{\gamma}(w_2, u_2)$ if and only if $w_1 = w_2$. Then $\tilde{\gamma}$ is the least semigroup congruence on $XT_a^b(1)$.*

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Free inverse dimonoids

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The notion of a dimonoid was introduced by J.-L. Loday in [1]. A nonempty set D with two binary associative operations \dashv and \vdash is called a *dimonoid* if for all $x, y, z \in D$ the following conditions hold:

$$\begin{aligned}(D_1) \quad & (x \dashv y) \dashv z = x \dashv (y \dashv z), \\(D_2) \quad & (x \vdash y) \dashv z = x \vdash (y \dashv z), \\(D_3) \quad & (x \dashv y) \vdash z = x \vdash (y \vdash z).\end{aligned}$$

If operations of a dimonoid coincide, it becomes a semigroup.

For the free inverse semigroup FI_X with a set X of free generators there exist different constructions. In [2], a new construction of the free inverse semigroup was proposed, in which elements of FI_X as shortest words in the classes of equivalent words over the alphabet $X \cup X^{-1}$ are described and an algorithm that transforms any word into its shortest equivalent form is produced. Using the definition of the free inverse semigroup from [2], we study a construction of the free inverse dimonoid and its properties. Other free dimonoid constructions have been studied, e.g., in [3]–[5].

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Derivations and automorphisms of locally matrix algebras

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Let \mathbb{F} be a ground field. An associative \mathbb{F} -algebra A is a *locally matrix algebra* if for each finite subset of A there exists a subalgebra $B \subset A$ containing this subset such that $B \cong M_n(\mathbb{F})$ for some n . A locally matrix algebra A is *unital* if it contains a unit 1.

Let $\text{Der}(A)$ be the Lie algebra of all derivations of the algebra A and let $\text{Inder}(A)$ be the ideal of all inner derivations. The factor algebra $\text{Outer}(A) = \text{Der}(A)/\text{Inder}(A)$ is called the algebra of outer derivations of A .

We prove the analog of the result of H. Strade for locally finite-dimensional locally simple Lie algebras.

Theorem 1. *Let A be a countable-dimensional locally matrix algebra. Then the Lie algebra $\text{Outer}(A)$ is not locally finite-dimensional.*

Let $\text{Aut}(A)$ and $\text{Inn}(A)$ be the group of automorphisms and the group of inner automorphisms of the algebra A . The factor group $\text{Out}(A) = \text{Aut}(A)/\text{Inn}(A)$ is called the group of outer automorphisms of A .

We determine dimensions of Lie algebras $\text{Der}(A)$ and $\text{Outer}(A)$ and orders of groups $\text{Aut}(A)$ and $\text{Out}(A)$, where A is a countable-dimensional locally matrix algebra.

Theorem 2. *Let A be a countable-dimensional locally matrix algebra over a field \mathbb{F} . Then*

$$\dim_{\mathbb{F}} \text{Der}(A) = \dim_{\mathbb{F}} \text{Outer}(A) = |\mathbb{F}|^{\aleph_0}.$$

Theorem 3. *Let A be a countable-dimensional locally matrix algebra over a field \mathbb{F} . Then $|\text{Aut}(A)| = |\text{Out}(A)| = |\mathbb{F}|^{\aleph_0}$.*

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Computer algebra, Gröbner bases and elimination in free algebras

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In the realm of symbolic computer algebra, which was called *symbolic artificial intelligence* back in the 1970's, Gröbner bases constitute one of major tools. The ability to perform elimination of variables in the context of free associative algebras and their factor-algebras makes Gröbner bases indispensable. However, in non-Noetherian algebras one has to discuss on the decidability of problems; a practical approach is to perform computations up to a specified *degree bound*.

A well-known freely available computer algebra system SINGULAR (www.singular.uni-kl.de) was developed in Kaiserslautern, Germany, with important contributions of algebraists from Kyiv. Two its' subsystems for non-commutative algebras, are presented. LETTERPLACE, released very recently, works with finitely presented associative algebras and offers unprecedented functionality, both for ring theory and for homological algebra. Notably, the newest release works not only over effective fields, but also over the ring \mathbb{Z} , which allows to compute information, valid in all possible characteristics.

Via the specialization to G -algebras (PBW-algebras, which are Noetherian domains) the subsystem PLURAL provides powerful algorithms, which are widely used in e.g. D -module theory and algebraic analysis. Two concrete computation sessions together with the results will be discussed.

The only way to *thank* the developers of free mathematical software is to *cite* this software in your articles, for instance like [1], [2], or [3].

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Міжнародна математична конференція,
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